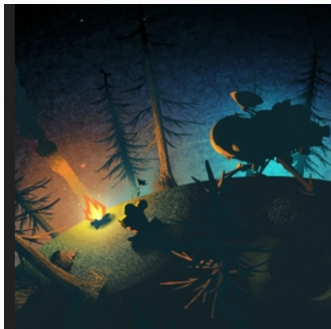


Games, graphs, and machines



October 8, 2025

Nim

- Piles of m&m's.
- Eat as many as you want (but at least 1) from **one** pile.
- The player who eats the last one wins.

Observations about nim

- Nim(7, 7) is P because of mirroring
- Nim(7, 9) is N
- Nim(4, 5, 4, 5) is P because of mirroring
- Nim(4, 5, 4, 5, 6) is N

Nim(2, 4, 9, 7, 11) ?

↳ on monday

Combining games

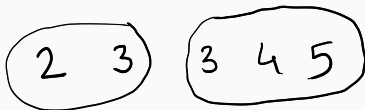
- $G + H =$ Play G and H side by side.
- A player may make a move in G or in H (but not in both).
- The player who cannot make a move loses.



Combining games

- $G + H =$ Play G and H side by side.
- A player may make a move in G or in H (but not in both).
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$$\text{Nim}(2, 3) + \text{Nim}(3, 4, 5) = \text{Nim}(2, 3, 3, 4, 5).$$



Nim + Chomp

Play Nim + Chomp



Addition rules

- ✓ • $N + P = P + N = N$
 - ✓ • $P + P = P$
 - $N + N$ could be either P or N .
-

Ex. ① Nim(1) is N
Nim(1) is N

Nim(1) + Nim(1) is P.

② Nim(1) is N
Nim(2) is N

Nim(1) + Nim(2) is N