

# Games, graphs, and machines

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## Sets are bags?

Use standard set notation to write down the sets represented by bags that Anand makes.

# Cardinality

Write down the cardinality of the following sets:

1.  $\{\}$

2.  $\{\{\}\}$

3.  $\{\{\}, \{\{\}\}\}$

# Cardinality

Write down the cardinality of the following sets:

1.  $\{\}$
2.  $\{\{\}\}$
3.  $\{\{\}, \{\{\}\}\}$



## Subset vs element

True or false?

1.  $\{\} \subset \{\}$

2.  $\{\} \in \{\}$

3.  $\{\} \in \{\{\}\}$

# Power set

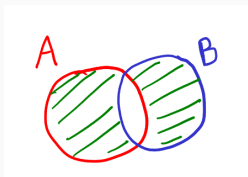
- What is the size of the power set of  $\{1, 2, 3, 4\}$ ?
- What about  $\{1, 2, 3, 4, 5\}$ ?
- What about  $\{1, \dots, 100\}$ ?

# Power set

- Suppose the size of  $A$  is  $n$ .
- What is the size of the power set of  $A$ ?
- Why?

# The symmetric difference

Suppose  $A$  and  $B$  are represented by the circles below. Using the operations of union, intersection, and difference, express the shaded set.



## Set operations

Let  $A = \{x^2 \mid x \in \mathbb{Z}\}$  and  $B = \{x^3 \mid x \in \mathbb{Z}\}$ . Write the smallest 5 elements of

1.  $A \cup B$
2.  $A \cap B$
3.  $A - B$
4.  $B - A$

## Set operations (continued)

Is the following true or false:  $|A - B| = |A| - |B|$ . If it is true, explain why. If it is not true, give a counter-example.