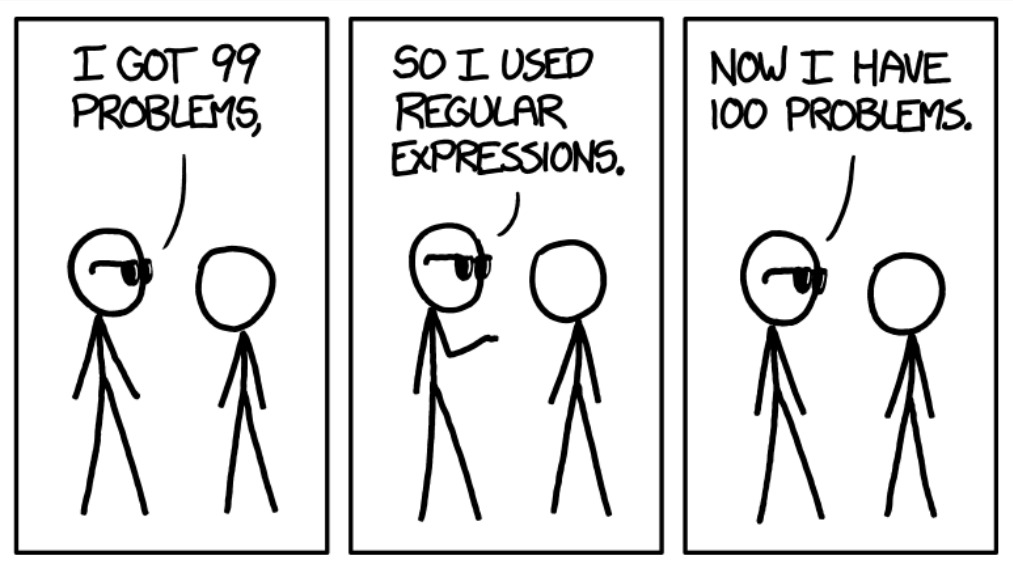
Games, graphs, and machines

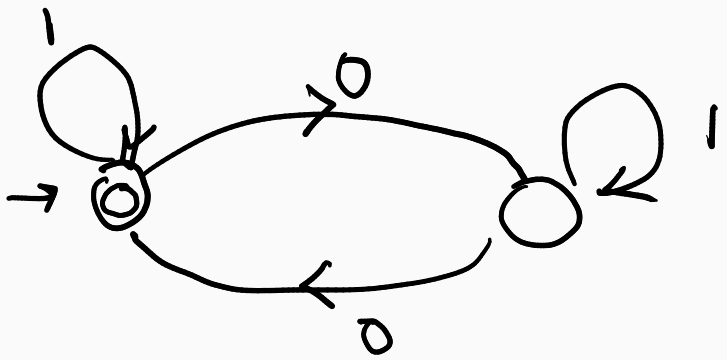


September 17, 2025

A regex

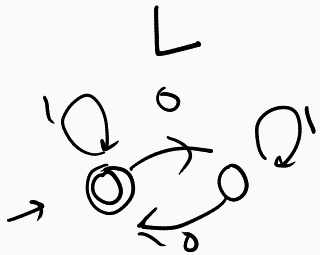
Let L be the language on $\{0, 1\}$ consisting of w that contain an even number of 0s. Can you construct a regex that describes L?

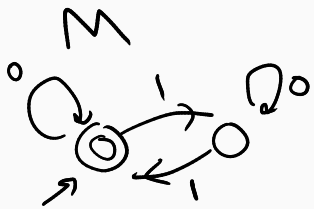
$$(1^*01^*01^*)^* \mid 1^*$$

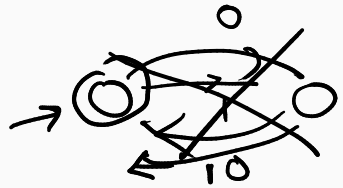


An automaton

Let M be the language on $\{0, 1\}$ consisting of w that contain an even number of 1s. Last time, we constructed an autamoton that describes L (and similarly M). Construct automata that describe $L \cup M$ and $L \cap M$.

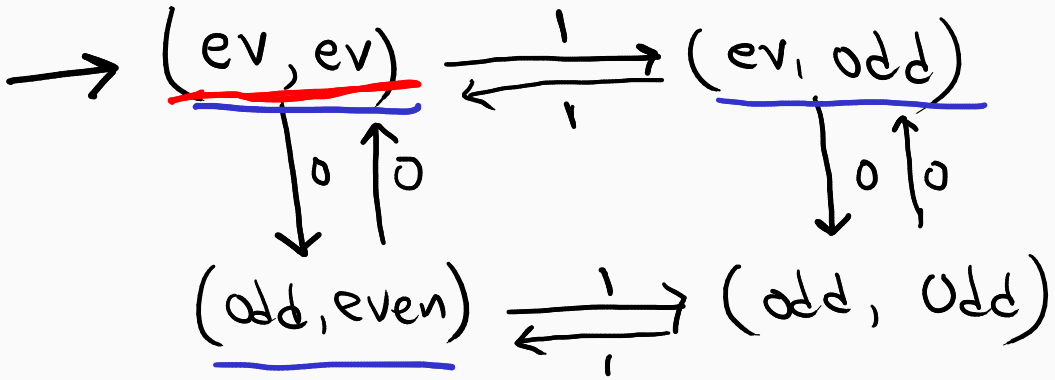






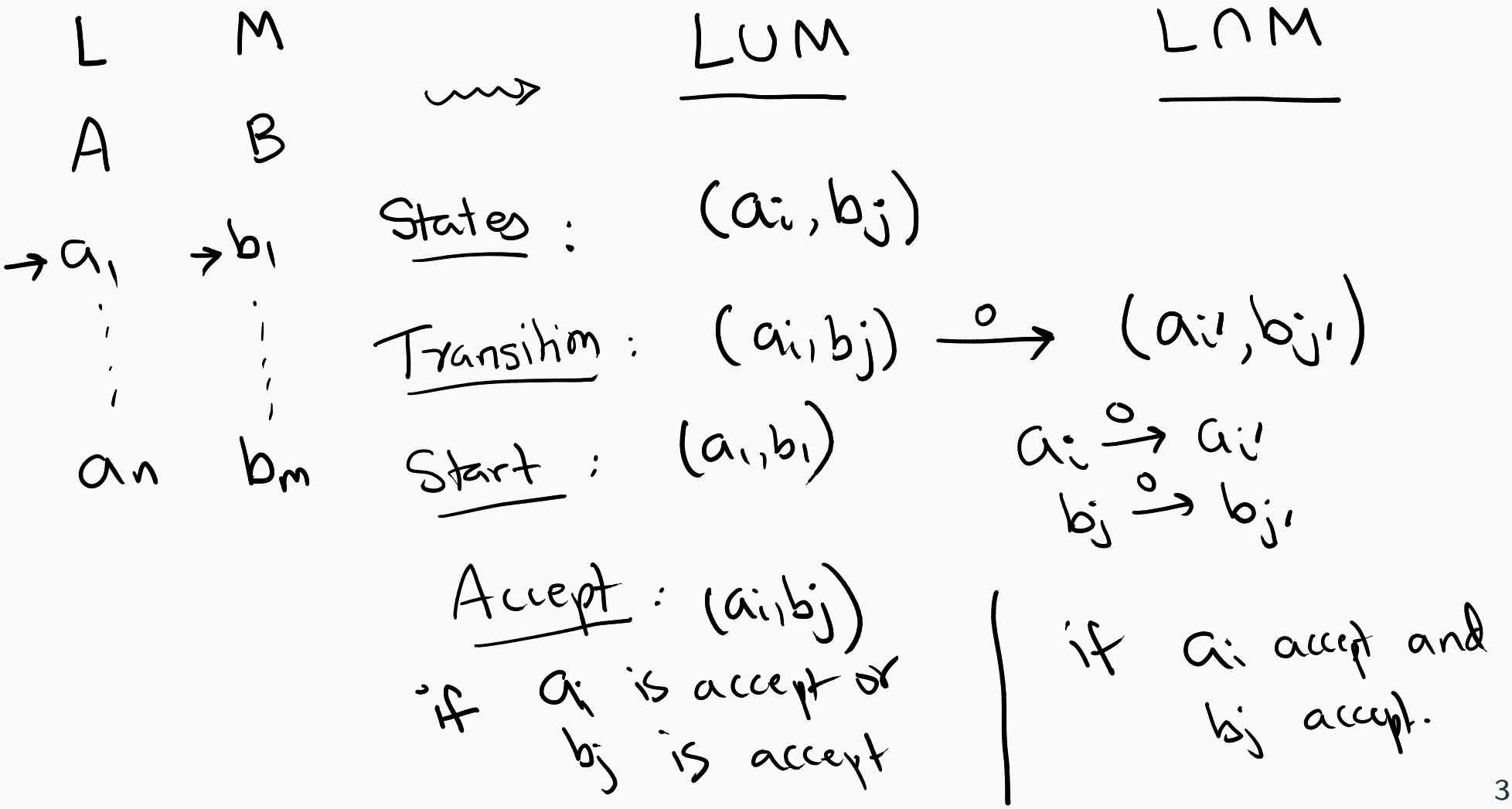
$L \cup M$

$L \cap M$



A generalisation

Given two languages L and M described by automatons A and B, construct automata that describe $L \cup M$ and $L \cap M$.



Regexes again?

Returning to L (even 0s) and M (even 1s), construct regexes that describe $L \cup M$ and $L \cap M$.

L: $(1^*01^*01^*)^*1^*$

M: $(0^*10^*10^*)^*0^*$

$L \cup M$:

$L \cap M$:

Regexes again?

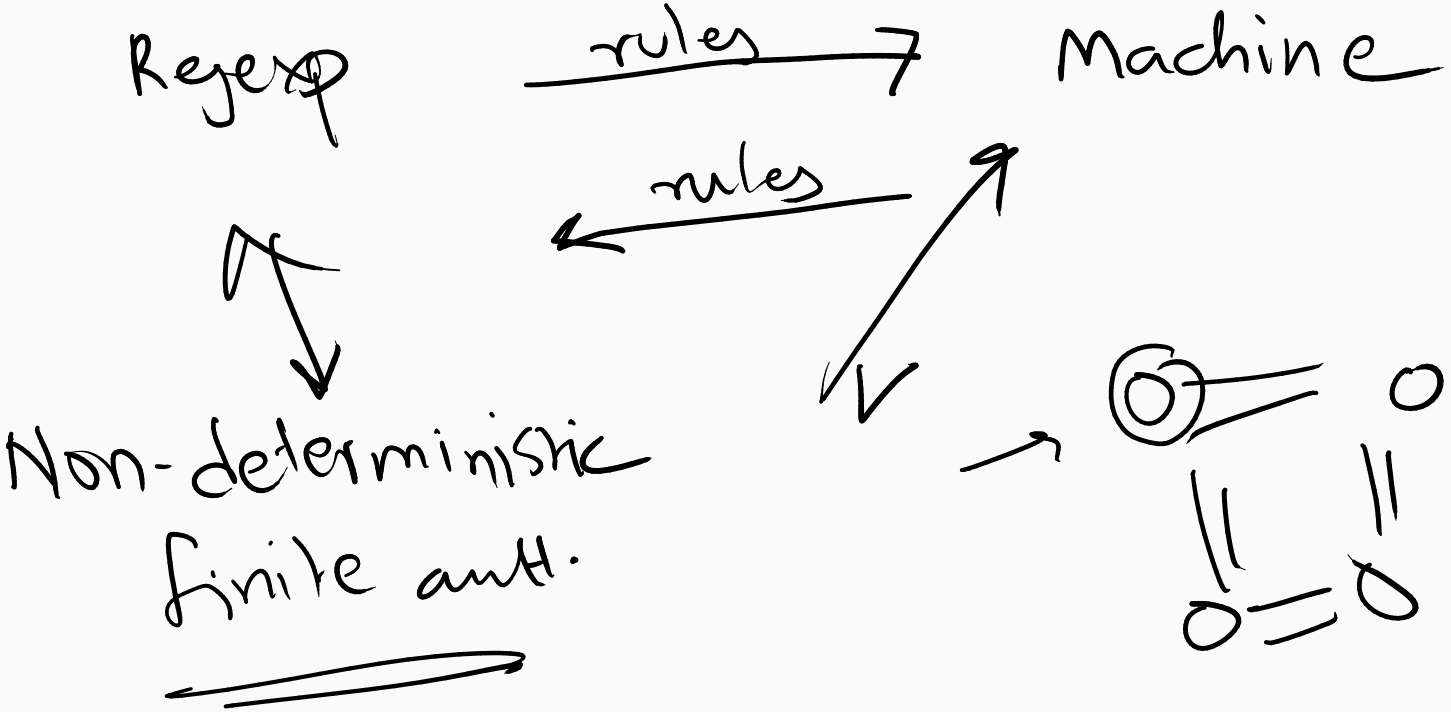
Returning to L (even 0s) and M (even 1s), construct regexes that describe $L \cup M$ and $L \cap M$.

For $L \cap M$:

$$(0(11)^*0 \mid (1 \mid 0(11)^*10)(0(0 \mid 1(11)^*10))^*(1 \mid 01(11)^*0))^*$$

Regexes and automatons are equivalent

Theorem Any language that can be described by a regex can also be described by an automaton, and vice-versa.



Better machines?

Searching for truth?

Let $L =$ (Mathematical) statements that are true. Is there an automaton (or a Turing machine) that describes L?

Gödel — not possible

The true reason why [no one] has succeeded in finding an unsolvable problem is, in my opinion, that there is no unsolvable problem. [. . .] We must know. We shall know!
– David Hilbert (1930)