

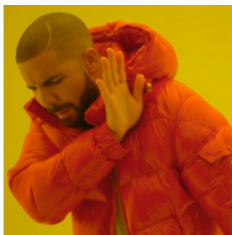
Games, graphs, and machines



October 15, 2025

Rules of Kayles

1. Knock down a single pin or two adjacent pins.
2. The player to knock down the last pin wins.



N/P
labels



Grundy
values

Analysing Kyles

$$K_7 = \text{mex}(K_6, K_5, K_1 + K_5, K_1 + K_4, K_2 + K_4, K_2 + K_3, K_3 + K_3)$$

Analysing Kyles

$$K_7 = \text{mex}(K_6, K_5, K_1 + K_5, K_1 + K_4, K_2 + K_4, K_2 + K_3, K_3 + K_3)$$

Better to just compute K_1, K_2, K_3, \dots in order.

Now play and win!

What is known?

Kayles nim-values through K_{83}

| K_n | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
|-------|---|---|---|---|---|---|---|---|---|---|----|----|
| 0+ | 0 | 1 | 2 | 3 | 1 | 4 | 3 | 2 | 1 | 4 | 2 | 6 |
| 12+ | 4 | 1 | 2 | 7 | 1 | 4 | 3 | 2 | 1 | 4 | 6 | 7 |
| 24+ | 4 | 1 | 2 | 8 | 5 | 4 | 7 | 2 | 1 | 8 | 6 | 7 |
| 36+ | 4 | 1 | 2 | 3 | 1 | 4 | 7 | 2 | 1 | 8 | 2 | 7 |
| 48+ | 4 | 1 | 2 | 8 | 1 | 4 | 7 | 2 | 1 | 4 | 2 | 7 |
| 60+ | 4 | 1 | 2 | 8 | 1 | 4 | 7 | 2 | 1 | 8 | 6 | 7 |
| 72+ | 4 | 1 | 2 | 8 | 1 | 4 | 7 | 2 | 1 | 8 | 2 | 7 |