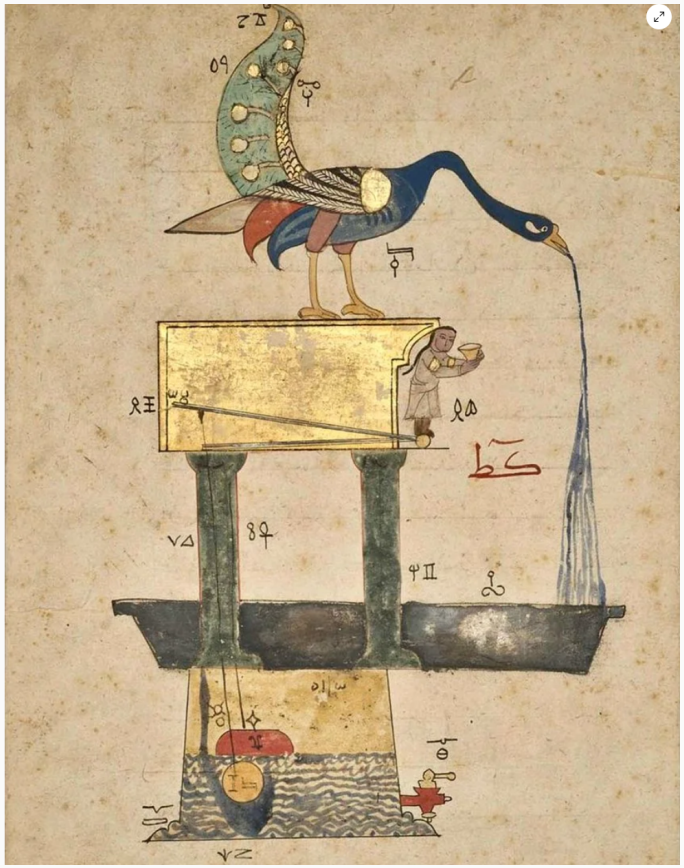
Games, graphs, and machines



September 16, 2025

Building trickier regexes

Find regular expressions that describe the following languages.

1. $\{w \mid 0$ and 1 alternate in $w\}$ (includes ε)
2. $\{w \mid$ every 0 in w has 1 on its left and on its right$\}$. (ind ε)
3. $\{w \mid w$ has an even number of 0s$\}$ (ind ε)

1. Examples

ε 0, 01, 010, 0101, 01010, ..., 0101010, ...

1, 10, 101, 1010, ...

$(01)^* \mid (10)^* \mid (01)^*0 \mid (10)^*1$

2. ....101....101....101....

$(1^*1011^*)^* \quad (1(1^*011^*)^* \mid 1^*)$

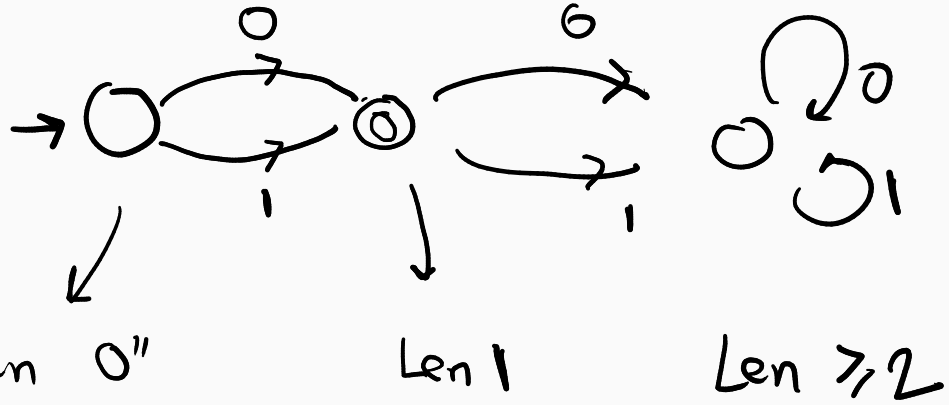
Building automata

Construct a DFA whose language is $\{0, 1\}$.

Del fin out.

Rej | Acc

ε 0,1



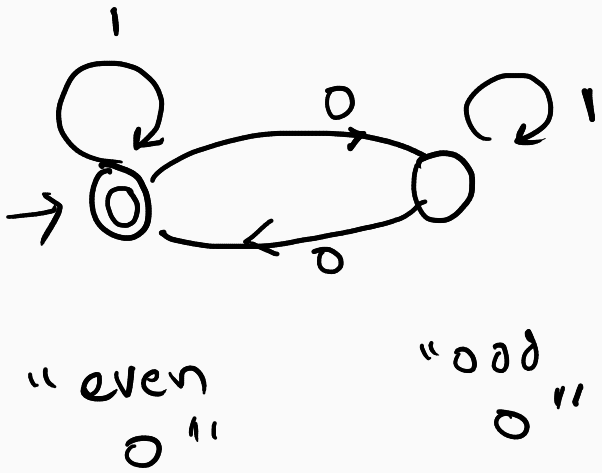
Building automata

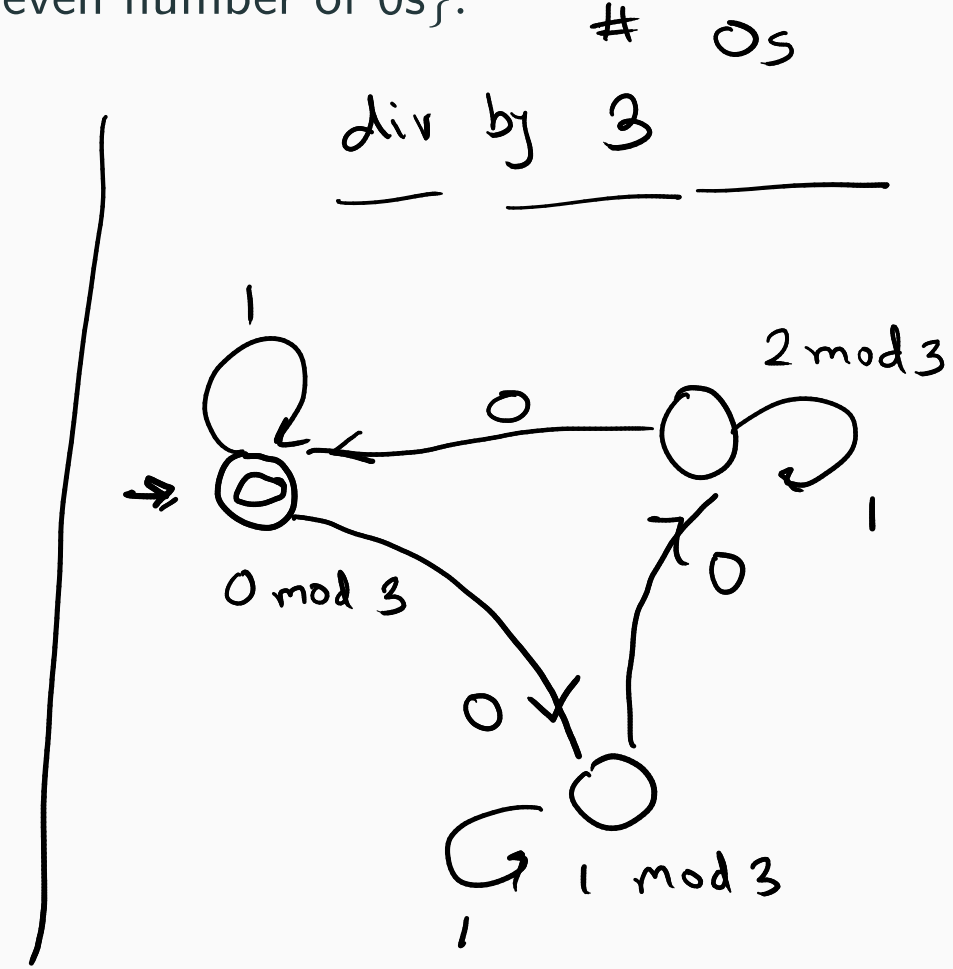
Construct a DFA with the same language as 01^*.

Building automata

Construct a DFA with the language

$$\{w \mid w \text{ has an even number of 0s}\}.$$

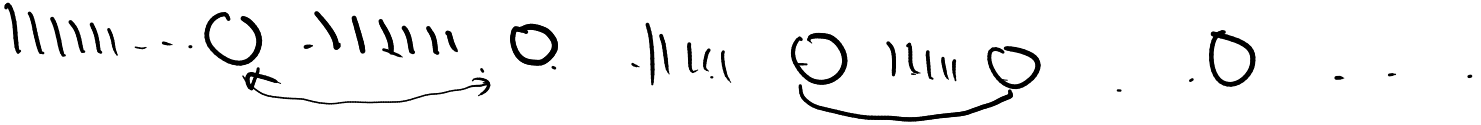




Regex for # 0's $\equiv 0 \mod 3$.

$$(1^*01^*01^*01^*)^* \mid 1^*$$

Regex for even 0s



$$(1^*01^*01^*)^* \mid 1^*$$

Building automata

Construct a DFA with the language

$$\{w \mid 0 \text{ and } 1 \text{ alternate in } w\}.$$

Even trickier languages

Can you find regexes or DFAs that describe the following languages?

1. $\{w \mid w \text{ has as many 0s as 1s}\}$.
2. $\{w \mid w \text{ is a palindrome}\}$.

↖ Turns out to be impossible.

0 . . . 0