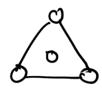
WEEK 12 WORKSHOP

MATH2301, SEMESTER 2, 2025

1. Grundy labels

The goal of this worksheet is to practice computing Grundy labels and using them to find winning moves.

Find the Grundy value of Chomp-the-graph for the graph



Using the Grundy value, find all possible winning moves for the first player.

A reminder of the rules: two players take turns removing either a vertex or an edge of G. When a vertex is removed, all edges adjacent to it are automatically removed. When an edge is removed, no other vertices (or edges) are removed. As usual, the player who cannot make a move loses.

For practice, do the problem with different graphs.

2. Stable equivalence

True or false:

(1) $\mathrm{Nim}(2,3) \sim \mathrm{Nim}(1,4)$.
(2) If $A \sim B$, then $A + C \sim B + C$ for any game C.
(3) If $A + C \sim B + C$, then $A \sim B$.
(4) If $A \sim B$, then the number of winning first moves in A is the same as the number of winning first moves in B.

3. Misere play (only for fun; not on the exam)

In *Misere play*, the player who makes the last move loses (so the player who cannot make a move wins).

(1) Try playing *nim* with the misere rule.
(2) How would you modify the rules of strategic (N/P) labelling for Misere play?
(3) Let us assign misere Grundy labels by the following rules: label the terminal nodes as 1; for non-terminal nodes, use the same rule: a node gets the mex of the labels of the children. With this labelling, convince yourself that a non-zero label means an N state and a zero label means a P state. But it is not true that the label of $G + H$ is the xor-sum of the labels of G and H. Find an example where this fails.

4. Strategic labelling for partisan games (also only for fun)

In partisan games, there are 4 possible outcomes: L (left has a winning strategy, irrespective of whether they go first or second), R (right has a winning strategy), N (first player has a winning strategy, irrespective of whether they are R or L), P (second player has a winning strategy). This means that we can label every game state as L, R, N, or P. Given a state, its R-children are the states reachable by R moves and L-children are the states reachable by L moves. Using R and L children, find the rules for strategic labelling. Here is start.

(1) If a state has no children (R or L), then label it P.
(2) If a state has R-children but no L-children, then label it R.
(3) If a state has L-children but no R-children, then label it L.
(4) If a state has an R-child labelled P/R and an L-child labelled P/L, then label it N.
(5) ... more ...

Date: October 22, 2025.

For impartial games, we saw that adding a P game does not change the outcome. Is this still true for partisan games?