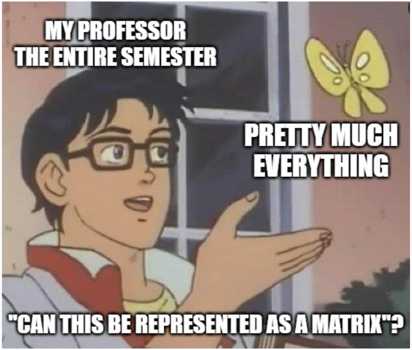
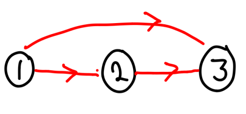
Games, graphs, and machines



August 12, 2025

Warm up

Find the adjacency matrix A.



Calculate A^2 and A^3.

A^k counts length k paths

Theorem
The (i, j) entry if A^k is the number of paths from vertex i to vertex j of length k.

Why does A^k count length k paths?

Theorem
The (i, j) entry if A^k is the number of paths from vertex i to vertex j of length k.

$k = 1$ Yes

$k = 2$ Let us see.

$$A^2_{i,j} = A_{i,1} \cdot A_{1,j} + A_{i,2} \cdot A_{2,j} + A_{i,3} \cdot A_{3,j}$$

Why does A^k count length k paths?

- $k = 3$

$$A^3_{i,j} = A^2_{i,1} \cdot A_{1,j} + A^2_{i,2} \cdot A_{2,j} + A^2_{i,3} \cdot A_{3,j}$$

Why does A^k count length k paths?

- $k = 4$

$$A^4_{i,j} = A^3_{i,1} \cdot A_{1,j} + A^3_{i,2} \cdot A_{2,j} + A^3_{i,3} \cdot A_{3,j}$$

Sum of powers

What do the entries of $A + A^2 + A^3 + A^4$ represent?

Acyclic graphs

We say that G is *acyclic* if it has no (directed) cycle.

Suppose G is acyclic and has 100 vertices. What can you say about A^{100}?

Longest path

Let G be a graph with adjacency matrix A.

Using A, how will you find the length of the longest possible path in G?