

Games, graphs, and machines

n	a(n)
0	1
1	1
2	3
3	19
4	219
5	4231
6	130023
7	6129859
8	431723379
9	44511042511
10	6611065248783
11	1396281677105899
12	414864951055853499
13	171850728381587059351
14	98484324257128207032183
15	77567171020440688353049939
16	83480529785490157813844256579
17	122152541250295322862941281269151
18	241939392597201176602897820148085023

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Chains

A *chain* in a poset is a sequence of elements a_1, \dots, a_n such that

$$a_1 \preceq a_2 \preceq \dots \preceq a_n.$$

The number n is the *length* of the chain.

Find a chain of length 3 in the subset poset of $\{1, 2, 3, 4\}$.

Maximal chains

- What could be the meaning of a *maximal chain*?
- Find a maximal chain in the subset poset of $\{1, 2, 3, 4\}$.

Maximal chains

- In the subset poset of $\{1, \dots, n\}$, all maximal chains have the same length. True/false?
- In any poset, all maximal chains have the same length. True/false?

Maximal chains and ranks

Let S be a finite poset.

Theorem

If all maximal chains in S have the length n , then there exists a rank function on S .

- Verify the theorem for the subset poset of $\{1, \dots, n\}$.
- Do you think the theorem is true? Why?

Converse?

Theorem

If all maximal chains in S have the length, then there exists a rank function on S .

The *converse* of “If A then B” is “If B then A”.

Do you think the converse of the theorem is true?