

# Games, graphs, and machines



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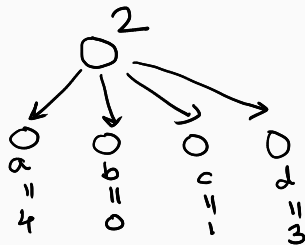
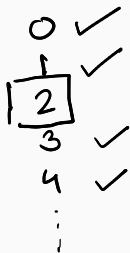
October 14, 2025

# Grundy value = Nim sum for non-nim games

How to compute the Grundy value?

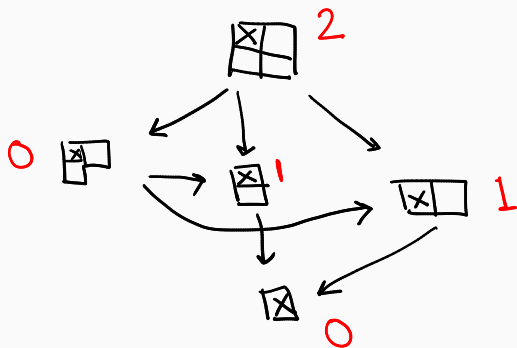
1. All sink states get 0.
2. Each state gets **mex** of its children.

Mex = “minimal excluded” (the smallest among the values not found among the children).



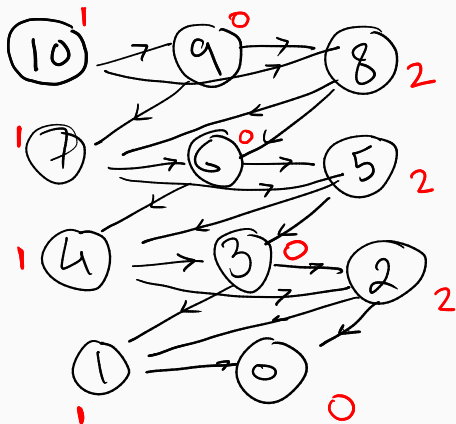
# Example

The Grundy value of  $2 \times 2$  Chomp.



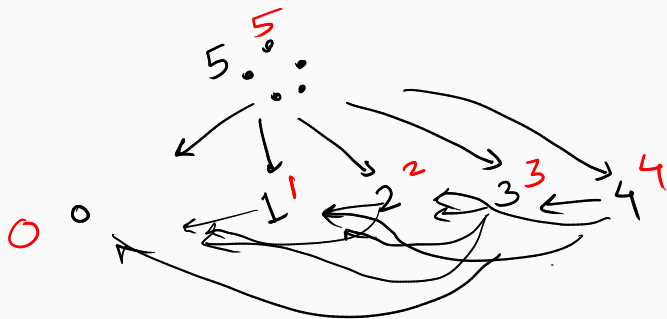
## Example

The Grundy value of Gobble starting with 10 berries and allowed to eat 1 or 2 berries.



## Example

The Grundy value of  $Nim(5)$ .



$Nim(n)$  has Grundy label  $n$ .

# Theorem

1. Grundy value of  $G = 0$  if and only if  $G$  is a  $P$ -game.
2. Grundy value of  $G \neq 0$  if and only if  $G$  is an  $N$ -game.
3. Grundy value of  $G + H = \text{Grundy value of } G \oplus \text{Grundy value of } H$ .

vs

3 for N/P :

$$N+P = N$$

$$P+N = N$$

$$P+P = P$$

$$N+N = ?$$

Can be resolved with Grundy's.

Nim(3,4,5)

has Grundy label 2

$$= \underset{3}{\text{Nim}(3)} \oplus \underset{4}{\text{Nim}(4)} \oplus \underset{5}{\text{Nim}(5)}$$

$$\begin{array}{r} 11 \\ 100 \\ 101 \\ \hline 010 \end{array}$$

# Application: how to play Chomp + Nim

Play (and win!)  $2 \times 2$ -Chomp + Nim(4, 7)

