

ASSIGNMENT 4 (DUE 22 AUGUST 2025)

MATH2301, SEMESTER 2, 2025

1. GENERAL REMARKS

- (1) The assignment is due on gradescope.
- (2) Please read the academic integrity policy for assignments.
- (3) The words “show” and “prove” are synonyms. You may not be used to writing formal mathematical proofs, which is OK. Write a justification in plain language that would convince the reader.
- (4) If you are having trouble with any of the points, come and discuss with me in office hours. It is part of my job to help you understand this stuff, so please use my time!

2. PROBLEMS

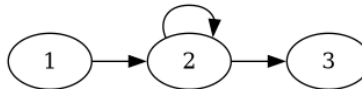
2.1. **Problem.** Consider a graph whose adjacency matrix is

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}.$$

Find the number of paths of length 4 from 1 to 3.

2.2. **Problem.** Find (without explicit calculation) an example of a 4×4 nonzero adjacency matrix such that all powers of this matrix beyond the 10th power are zero.

2.3. **Problem.** Let G be the directed snail



Let A be the adjacency matrix of G . Describe all positive powers of A .

2.4. **Problem.** This is an exploratory problem.

We can classify graphs according to how quickly the entries in the powers of the adjacency matrix grow. Denoting by A the adjacency matrix, we have

- (1) graphs such that $A^n = 0$ for large enough n .
- (2) graphs such that the entries of A^n are bounded by a constant,
- (3) graphs such that the entries of A^n are bounded by a polynomial function of n ,
- (4) graphs such that the entries of A^n grow exponentially as n grows.

Find examples of graphs in each class.

Given a graph, is there a way to easily tell where falls among the classes above?