WEEK 10 WORKSHOP

MATH2301, SEMESTER 2, 2025

1. Warm-up

1.1. **Problem.** Play some of the games we have learned: the subtraction game with different starting values and parameters, chomp, and nim.

1.2. **Problem.** Remember the rules of P vs N labelling.

(1) If a state is labelled P, then which player has a winning strategy?
(2) If a state is labelled N, then which player has a winning strategy?
(3) If a state is a sink state (no outgoing arrows), we label it ______.
(4) If a state has an outgoing arrow to a P state, we label it ______.
(5) If all outgoing arrows of a state are N, we label it ______.

1.3. **Problem.** Draw the game graph of some small games and label the game states as P or N. For example:

(1) Eat 1 or 2 or 3 berries starting with 7 berries.
(2) Eat any power of 2 (including $2^0 = 1$) berries starting with 15 berries.
(3) 2×2 and 2×3 chomp.

2. Adding two games

2.1. **Problem.** Explain what it means to add two games.

2.2. **Problem.** Find two N games G_1 and G_2 such that $G_1 + \mathrm{Nim}(2,3)$ and $G_2 + \mathrm{Nim}(2,3)$ have different outcomes (N vs P).

3. Wyt rooks

The game of Wyt rooks is played as follows. We start with two rooks placed on two squares of a chessboard. A move consists of moving one of the rooks any positive number of squares downward or leftward (but not both). Rooks can occupy the same square and move past each other. As usual, the player who cannot make a move loses.

3.1. **Problem.** Play this game with your group and determine some winning and losing positions.

3.2. **Problem.** This game is actually a nim game in disguise. Can you figure out which nim game and how?

4. Euclid's game

Euclid's game is defined as follows. The starting position is a pair of positive integers (a, b). A move consists of subtracting a non-zero multiple of the smaller number from the larger number, ending up again with a pair of *positive* integers. In particular, any position of the form (n, n) is a P position.

4.1. **Problems.** Play this game with your group and determine some other non-trivial P and N positions.
Bonus/challenge: Can you find a pattern in who has a winning strategy?

Date: October 6, 2025.