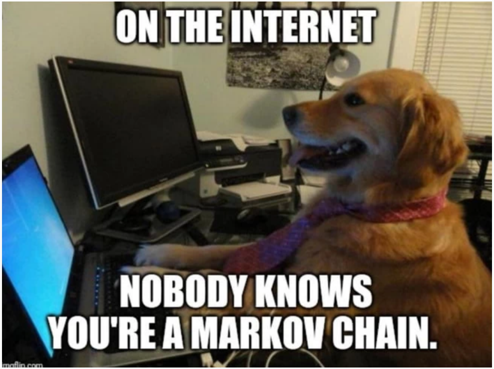
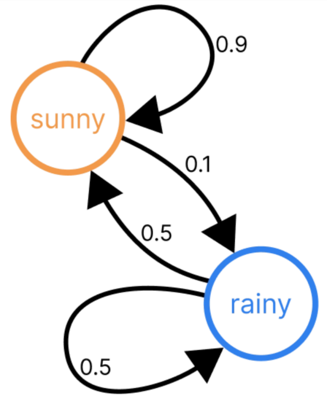
Games, graphs, and machines



August 25, 2025

Warm up

Write the transition matrix A for the following Markov chain.



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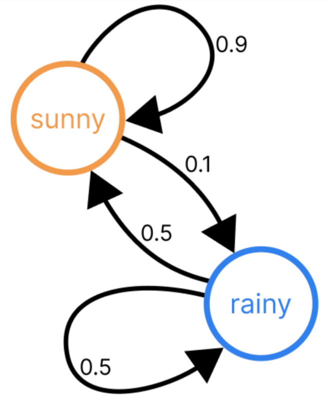


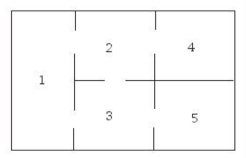
Calculate A^2. What do entries of A^2 represent?

Why does kth power represent k-step probabilities?

$$A^2_{i,j} = A_{i,1} \cdot A_{1,j} + A_{i,2} \cdot A_{2,j}.$$

A maze

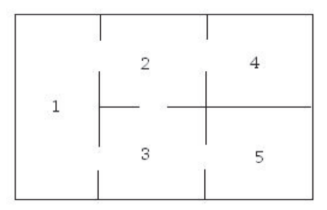
A maze used for training rats has the following shape.



Suppose that at every stage, the rat picks a door at random and goes through that door. Write the Markov chain and the transition matrix.

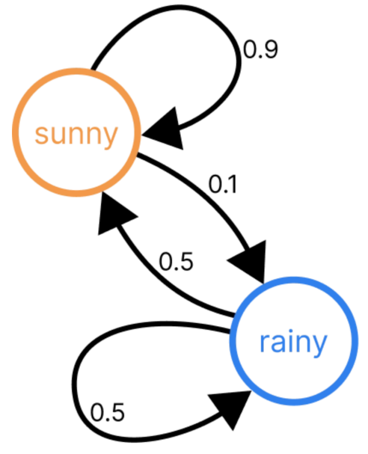
Powers of the maze

See if the powers of the matrix converge and interpret the result.



Powers of the weather

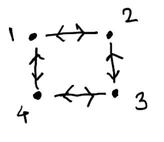
See if the powers of the matrix converge and interpret the result.



A random walk

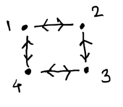
Let G be a directed graph. A random walk on G consists of walking on G by the following simple rule: pick an outgoing edge at random and follow it.

Write the transition matrix for the random walk on



Powers of the random walk

See if the powers of the matrix converge and interpret the result.



Understanding large powers

It turns out that the sunny/rainy matrix A can be written as

$$A = EDE^{-1},$$

where $E = \begin{pmatrix} 1 & 1 \\ 1 & -5 \end{pmatrix} \quad D = \begin{pmatrix} 1 & 0 \\ 0 & 0.4 \end{pmatrix} \quad E^{-1} = \begin{pmatrix} 5/6 & 1/6 \\ 1/6 & -1/6 \end{pmatrix}.$

Use this to understand A^k for large k.

When do large powers converge?

Suppose $A = EDE^{-1}$, where D is diagonal. When will A^k converge (to a matrix with finite entries) as k grows?