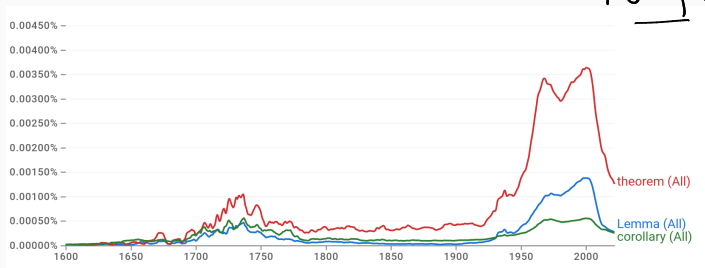


# Games, graphs, and machines

Pumping  
lemma



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September 30, 2025

# Pumping lemma

Feed a long word to a  
small machine  $\Rightarrow$  must be

a loop

010101111110

01 010111 010111 1110

the loop can be  
repeated  $n$  times  
for any  $n$ .

# Pumping lemma 1

Use the pumping lemma to show that  $L = \{0^n 1^n \mid n \geq 0\}$  is not regular.

Pumping lemma



Using loops to break machines.

# How to think about the pumping lemma?

**Machine** I can recognise  $L$ .

**You** How many states do you have?

**Machine** 212

**You** What would you do to  $0^{300}1^{300}$ ?

**Machine** Accept, obviously.

**You** And as you are reading that, you loop in the first 212 bits?

**Machine** Of course.

**You** If you double that loop, do you still accept?

**Machine** Yes, I guess. ✓

**You** Checkmate!



## Pumping lemma 2

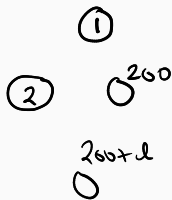
Use the pumping lemma to show that  $L = \{\text{Palindrome}\}$  is not regular.

You : How many states?

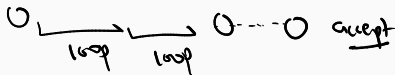
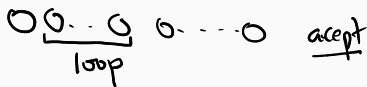
Machine : 100

You : .....

↪ Same forwards  
& back



0110 no nothing.



③

$0^{200} 1 0^{200}$

accepts.

$0 \dots 0 1 0 \dots 0$   
└──┬──┘  
loop

$0 \dots 0 | 0 \dots 0$   
└──┬──┘ └──┬──┘  
loop loop

accept

↘  
Not a palindrome.

## Pumping lemma 3

Use the pumping lemma to show that  $L = \{0^n 1^m \mid n \geq m\}$  is not regular.

M: I can do this.    You: How many states?

M: 37    You:

①  $0^{38} 1^{39}$  rejects.

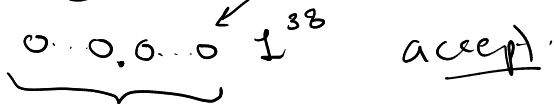
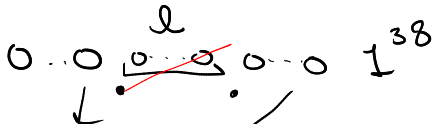
$0 \dots 0 \xrightarrow{\text{loop}} 0 \dots 0 1^{39}$  rej

$0 \dots 0 \text{lllll} 0 \dots 0 1^{39}$  rej.

but should have been accepted.

②

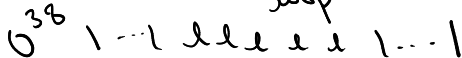
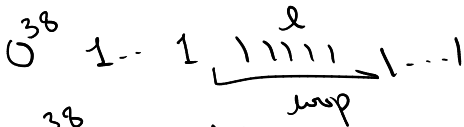
$0^{38} 1^{38}$  accept.



Smaller than  $38$

③

$0^{38} 1^{38}$  accept.



but should have rejected!  
↑  
accepts

## Pumping lemma 4

Use the pumping lemma to show that  $L = \{ww \mid w \text{ any string}\}$  is not regular.

Suppose there is  $M$  with  $n$  states.

Consider:  $0^{n+1}00 \quad 1 \quad 0^{n+1}00 \quad 1$

Has to be a loop as  $M$  reads  $n+1$  0's.

$$0^{n+1}00 = 0^m \underbrace{0^l}_{\text{loop}} 0^r \quad 1 \quad 0^{n+1}00 \quad 1$$

$0^m 0^l 0^l 0^r \quad 1 \quad 0^{n+1}00 \quad 1$  is accepted  
but should be rejected.