QUIZ 6 (MATH2301, 2025)

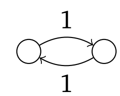
Name: ____________________

UID: ____________________

Justifications are not required in any of the questions.

(1) (4 points) State true or false.

(a) Let A be the transition matrix of a the following Markov chain:



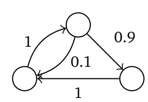
Then A^5 is the identity matrix.

☐ True ☐ False .

(b) Let A be the transition matrix of a Markov chain with a strongly connected graph. Then $\lim_{n\to\infty} A^n$ must exist.

☐ True ☐ False .

(c) The Markov chain below satisfies the Perron-Frobenius property

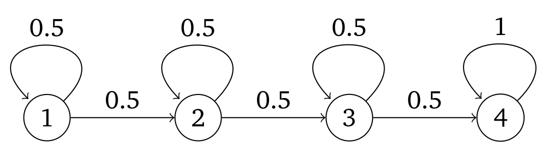


☐ True ☐ False .

(d) Let A be the transition matrix of a Markov chain. If $\lim_{n\to\infty} A^n$ exists, then all its rows are equal.

☐ True ☐ False .

(2) (3 points) Let A be the transition matrix of the Markov chain.



$A^2_{1,1} =$ $A^2_{1,4} =$ $A^3_{1,4} =$.

(3) (3 points) In the same Markov chain above, let $B = \lim_{n\to\infty} A^n$ (this exists).

$B_{4,4} =$ $B_{1,1} =$ $B_{3,4} =$.

1. SOLUTIONS

(1) True or false
   (a) False (it has 1s on the anti-diagonal and 0s on the diagonal)

   (b) False (the above example is a counter-example)

   (c) True

   (d) False. For example, consider our win-by-2 game.

(2)
- $A^2_{1,1} = 0.5^2 = 0.25$
- $A^2_{1,4} = 0$
- $A^3_{1,4} = (0.5)^3 = 0.125$.

(3)
- $B_{4,4} = 1$
- $B_{1,1} = 0$
- $B_{3,4} = 1$.

  This is probably the least obvious. But it is obvious that $B_{3,1} = B_{2,1} = 0$. And by the same logic as $B_{4,4} = 0$, we get $B_{3,3} = 0$. Since the rows have to sum up to 1, we must have $B_{3,4} = 1$.
  A more direct way to see it is to observe that

$$A^n_{3,4} = 0.5 + 0.5^2 + \cdots + 0.5^{n-1}.$$

  So $B_{3,4} = 0.5 + 0.5^2 + \cdots = 1$.