WEEK 4 WORKSHOP
MATH2301, SEMESTER 2, 2025

1. Matrix multiplication

1.1. **Warm-up.** Take two 2×2 matrices A and B of your choice and calculate AB. Try to find a pairs such that

(1) $AB \neq BA$

(2) $AB = 0$ but $A \neq 0$ and $B \neq 0$ (here 0 means the 0 matrix, which is the matrix all of whose entries are 0.)

1.2. **The identity matrix.** The $n \times n$ identity matrix has 1's on the diagonal and 0's everywhere else. It is denoted by I_n or $I_{n\times n}$ or just I if the n is clear from the context. Convince yourself that for any $n \times n$ matrix A, we have

$$A \cdot I_n = I_n \cdot A = A.$$

2. Adjacency matrices

2.1. **Poset graphs.** Given a poset S, we can construct a directed graph $G(S)$ as follows. The vertices of $G(S)$ are the elements of S, and we draw an edge $a \to b$ if $a \leq b$.

Draw the graph $G(S)$ and write the adjacency matrix for the following S

(1) $S = \{1, 2, 3\}$ with the usual order.

(2) S is the divisor poset of 12.

3. Path counting

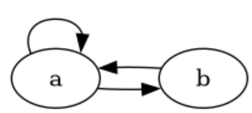
3.1. **Warm-up.** Interpret $A = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ as the adjacency matrix of a graph. Calculate A^n for $n = 1, 2, 3, 4, \cdots$. Can you explain what you see in terms of path counting?

3.2. **Poset matrices.** Let S be the poset and A the adjacency matrix from the problem above on poset graphs.

(1) For $S = \{1, 2, 3\}$, find $(A - I)^3$. Can you explain the result?

(2) For S equal to the divisor poset of 12, what do you think is the smallest positive n such that $(A - I)^n = 0$?

3.3. **Rabbits?** Consider the following graph.



Calculate for a few values of k the number of length k paths from a to itself. Can you find (and perhaps prove!) a pattern?

Date: August 12, 2025.