ASSIGNMENT 10 (DUE ON 17 OCTOBER 2025)

MATH2301, SEMESTER 2, 2025

1. General remarks

(1) The assignment is due on gradescope.
(2) Please read the academic integrity policy for assignments. Remember that if you want an extension, you must ask at least 24 hours ahead of the deadline.
(3) The words "show" and "prove" are synonyms. You may not be used to writing formal mathematical proofs, which is OK. Write a justification in plain language that would convince the reader.
(4) If you are having trouble with any of the points, come and discuss with me in office hours. It is part of my job to help you understand this stuff, so please use my time!

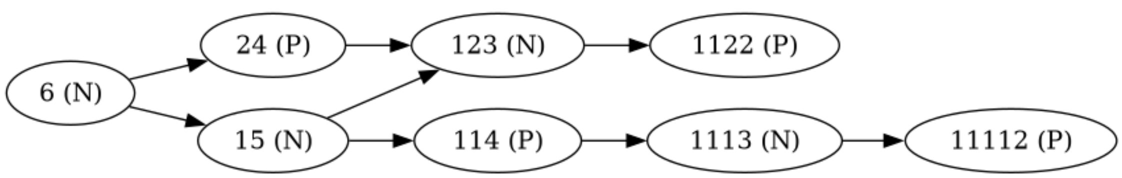
2. Problems

2.1. **Problem.** Grundy's game is defined as follows. The starting position is some number of piles of berries. A move consists of taking *one* pile of berries, and dividing it into two non-empty piles of *unequal sizes*. For example, a pile of size 4 can only be split into $1/3$, whereas a pile of size 5 can be split either as $1/4$ or as $2/3$. The person who can't make a move loses: at this point, the game board will only have piles of sizes either 1 or 2.

Draw the game graph starting at the position 6, labelling each position as either N or P. Consider the piles to be unordered; that is, a position such as $(1,4)$ is considered to be the same as the position $(4,1)$.

No justification is required.

2.1.1. *Solution.* Here is the answer.



2.2. **Problem.** The game *poset chomp* is played as follows. We start with a finite poset S. A move consists of removing an $a \in S$ together with all $b \in S$ with $a \preceq b$.

Let S be the divisor poset of 12 except the integer 1. So $S = \{2,3,4,6,12\}$ and $a \preceq b$ if a divides b. Using strategic labelling on the game graph, determine if the following positions are N or P:

(1) $\{3\}$
(2) $\{2,4\}$
(3) $\{2,3,4\}$
(4) $\{2,3,4,6,12\}$

2.2.1. *Solution.*

(1) N
(2) N
(3) N
(4) N

2.3. **Problem.** Determine if the following nim games are P or N.

(1) Nim$(3,4,5)$
(2) Nim(m,m,n) where $m,n>0$.
(3) Nim(m,m,m,m) where $m>0$.
(4) Nim(m,n) where $m\neq n$.
(5) Nim$(m,2m,4m,8m)$ where $m>0$.

2.3.1. *Solution.* The most direct way is to just see if the nim-sum is zero or non-zero.

(1) N
(2) N
(3) P
(4) N
(5) N

2.4. **Problem.** The game of nimble is played on a line of squares labelled $0,1,2,3,\ldots$. There are n coins placed on the squares, with perhaps more than one coin on a single square. A move consists of taking one of the coins and moving it to any square on the left, possibly moving over some coins, and possibly onto a square already containing one or more coins. The players alternate moves and the games end when all coins are on the square labelled 0.

(1) Convince yourself (but do not turn in) that this game is just nim in disguise.
(2) Suppose the coins are initially placed on the squares $4,8,11$. This is an N position. Find all possible winning moves. Express your answer as a triple (a,b,c) denoting the positions of the coins. For example, $(3,8,11)$ is a valid move (first coin moved from 4 to 3). And so is $(4,0,11)$, but not $(3,0,11)$.

2.4.1. *Solution.*

(1) This game is nim with as many piles as the number of coins. If a coin is on number x, we treat it as representing a pile of x stones in nim. Then the moves in this game correspond exactly to the moves of nim.
(2) The nim-sum of Nim$(4,8,11)$ is $100\oplus 1000\oplus 1011=111$. We have $100\oplus 111=11$, which is 3. So one winning move is to move the 4 to a 3. We have $1000\oplus 111=1111$, which is 15. Since it is greater than 8, there is no winning move on the 8-pile. We have $1011+111=1100$, which is 12. Again, greater than 11 means that there is no winning move on the 11-pile.

    So the only winning move is to move the coin on position 4 to position 3.