

**QUIZ 3**  
**MATH2301, 2025**

No justification required in any question.

(1) (4 points) Select true or false:

(a) Let  $S = \{1, 2, 3, 4\}$ . The relation  $\{(a, b) \in S \times S \mid a - b \geq 1\}$  is a partial order.

True

False

(b) Let  $S = \{1, 2, 4, 8\}$  with the divisibility relation ( $a \leq b$  if  $a \mid b$ ). Then this is a total ordering on  $S$ .

True

False

(c) If  $\leq$  is a partial order on a finite set  $S$ , then every element of  $S$  has at least one immediate successor.

True

False

(d) If  $\leq$  is a partial order on a finite set  $S$ , then every element of  $S$  has at most one immediate successor.

True

False

(2) (3 points) Draw the Hasse diagram of a poset with at least 5 elements that has **exactly one maximum element** and **exactly two minimal elements**.

(3) (3 points) Let  $S$  be the subset of  $\text{Pow}(\{1, \dots, 10\})$  consisting of all subsets whose size is even. So  $\{1, 2\} \in S$ , but  $\{1, 2, 3\} \notin S$ . For  $A, B \in S$ , declare  $A \leq B$  if  $A \subseteq B$ .

Let  $f$  be a rank function on  $S$  such that  $f(\emptyset) = 0$ . What is  $f(\{1, 2, 3, 4, 5, 6\})$ ?

$f(\{1, 2, 3, 4, 5, 6\}) =$

1. SOLUTION

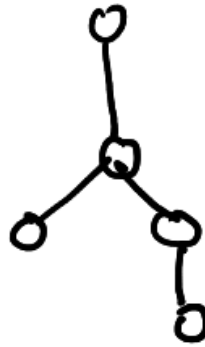
(1) (a) False (not reflexive)

(b) True

(c) False (maximal elements have no successors)

(d) False (there may be more than one immediate successors)

(2) Many possibilities. For example:



(3)  $f(\{1, 2, 3, 4, 5, 6\}) = 3$  because we have the chain of immediate successors  $\{1, 2\} \leq \{1, 2, 3, 4\} \leq \{1, 2, 3, 4, 5, 6\}$ .