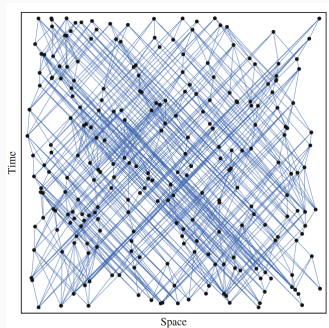


Games, graphs, and machines



causality poset

$a \preceq b$ if

a could cause b

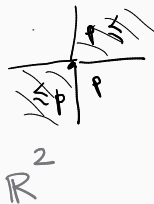
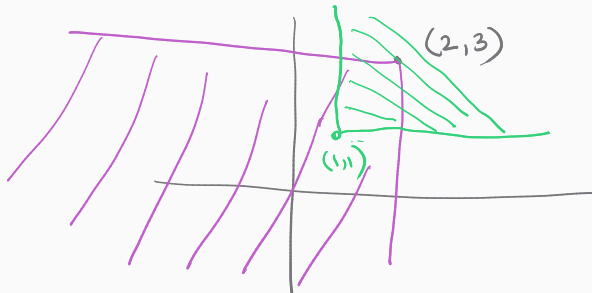
August 5, 2025

Product poset

Let \leq be the usual order on \mathbb{R} . Define \preceq on $\mathbb{R} \times \mathbb{R}$ by

$$(a, b) \preceq (c, d) \text{ if } \begin{matrix} a \leq b \text{ and } c \leq d \\ a \leq c \text{ and } b \leq d \end{matrix}$$

1. Give an example of two incomparable elements under \preceq .
2. Plot all elements that are $\preceq (2, 3)$.
3. Plot all elements (x, y) with $(1, 1) \preceq (x, y) \preceq (2, 3)$.



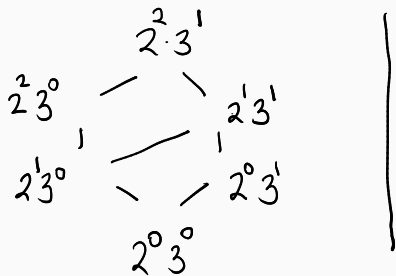
Divisor poset as product poset

Draw the Hasse diagrams of

1. the divisor poset of 12

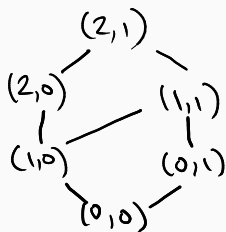
2. the product poset of $\{1, 2, 3\}^{\leq}$ and $\{1, 2\}^{\leq}$.

$$\{0, 1, 2\}^{\leq}, \{0, 1\}^{\leq}$$



$$2^a 3^b \mid 2^c 3^d$$

$$\Leftrightarrow a \leq b \text{ and } c \leq d$$



$$\text{Div. poset of } 36 = 2^2 \cdot 3^2$$

$$= \text{Product poset of } \{0,1,2\}^{\leq} \text{ and } \{0,1,2\}^{\leq}$$

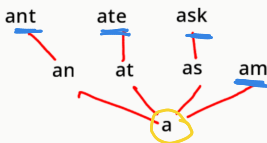
$$\text{Div. poset of } 2^3 \cdot 3^2 \cdot 5^7 =$$

$$\text{Product poset of } \{0,1,2,3\}, \\ \{0,1,2\}, \\ \{0,1,\dots,7\}$$

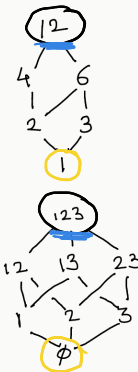
Max/min

0

In all the examples so far, identify the maximum (if it exists), the minimum (if it exists), all maximal elements, all minimal elements.

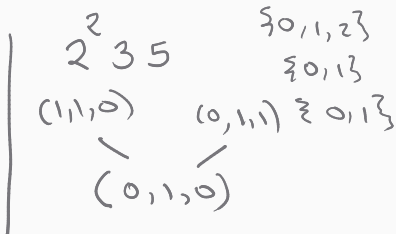
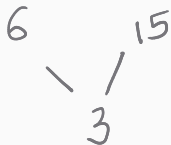


Product poset \mathbb{R}^2



Immediate successors

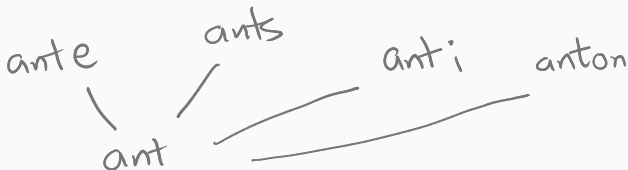
Let S be the divisor poset of 60. What are the immediate successors of 3?



Immediate successors

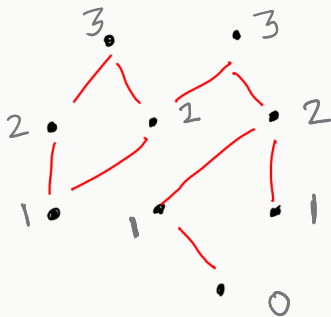
Let S be the poset of words with \preceq given by prefix.

- What are the immediate successors of “ant”?
- What is an element that succeeds “ant” but is not an immediate successor?



Rank function

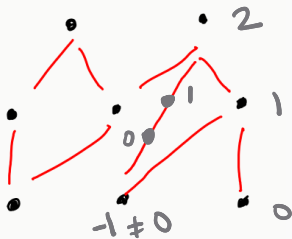
Find a rank function on the following poset.



Rank function?

Find a rank function on the following poset.

None



When does a rank function exist?

Under what conditions does a rank function exist on a poset?

Post on Canvas!