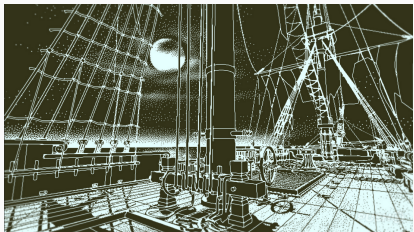


Games, graphs, and machines



Return of the
Obra Dinn

October 13, 2025

How to win at nim?

Find the nim-sum of Nim(4, 10, 12).

$$10 \xrightarrow{1/2} 5 \xrightarrow{-1/2} 2 \rightarrow 1 \rightarrow 0$$

4 in binary

10 →

12 →

8+4

1	0	0	
1	0	1	0
1	1	0	0
0	0	1	0

Non-zero

⇓

2

\oplus = xor

$$0 \oplus 0 = 0$$

$$1 \oplus 0 = 1$$

$$0 \oplus 1 = 1$$

$$1 \oplus 1 = 0$$

Nim sum is 10 in binary (2 in dec)

Nim (4, 10, 12) \Rightarrow 10

$$\begin{array}{r} 110 \\ 1010 \\ 1100 \\ \hline 0000 \end{array}$$

$$100 \oplus 1010 \oplus 1100 = 10$$

4, 10, 12 \rightarrow 4, 10, 12, 2



$$\begin{array}{r} 100 \\ \oplus \\ 10 \\ \hline 110 \\ \downarrow \\ 6 \end{array}, 10, 12$$

$$\underline{\underline{\oplus 10}}$$

4, 10, 12 \rightarrow 6, 10, 12

4, $\begin{array}{r} 1010 \\ 10 \\ \hline 1000 \end{array}$, 12 \rightarrow 4, 8, 12

4,10,12 \longrightarrow 4,8,12 \longrightarrow 8,12

8,12 \longrightarrow 8,8

$$\begin{array}{r} 1000 \\ 1100 \\ \hline 0100 \end{array}$$

$$\begin{array}{r} 1000 \\ 100 \\ \hline 1100 \times \end{array}$$

$$\begin{array}{r|l} 1100 & \\ 100 & \\ \hline 1000 & \checkmark \end{array}$$

\searrow
8,5

$$\begin{array}{r} 1000 \\ 101 \\ \hline 1101 \end{array}$$

$$\begin{array}{r} 1000 \\ 1101 \\ \hline 0101 \end{array} \checkmark$$

$$\begin{array}{r} 101 \\ 1101 \\ \hline 1000 \times \end{array}$$

\swarrow
5,5

5, 7, 3 has nim sum = 1

4, 7, 3

5, 6, 3

5, 7, 2

$$\begin{array}{r} 11 \\ - 1 \\ \hline 10 \end{array}$$

2, 6, 3

$$\begin{array}{r} 10 \\ 110 \\ - 11 \\ \hline 111 \end{array}$$

~~$$\begin{array}{r} 10 \\ 111 \\ - 101 \\ \hline \end{array}$$~~

2, 1, 3

~~$$\begin{array}{r} 11 \\ 106 \\ - 106 \\ \hline \end{array}$$~~

$$\begin{array}{r} 110 \\ 111 \\ - 101 \\ \hline 001 \end{array} \checkmark$$

2, 1, 1

1, 1, 1

How to win at nim?

Find the nim-sum of $\text{Nim}(4, 10, 12)$.

Is this an N -position or a P -position?

How to win at nim?

Find the nim-sum of $\text{Nim}(4, 10, 12)$.

Is this an N -position or a P -position?

Find all winning moves.

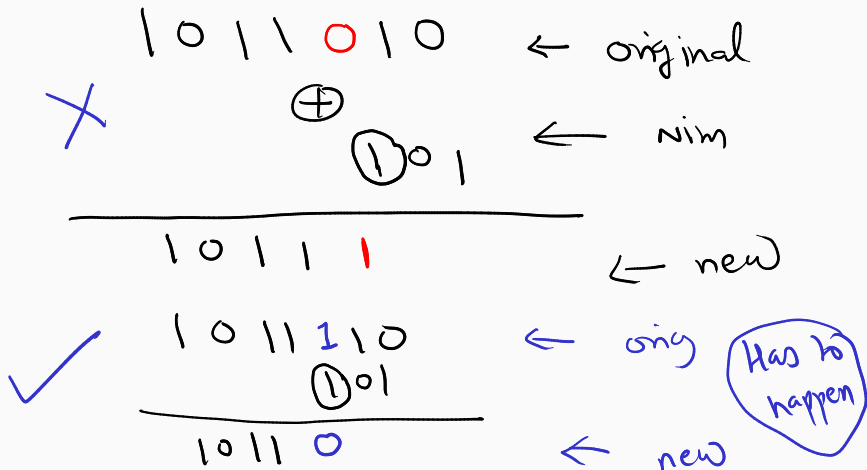
Continue playing

Convince yourself that from a zero nim-sum, all moves result in a non-zero nim-sum.

$$\begin{array}{l} \textcircled{3} \quad 5 \quad 3 \quad 5 \\ 3 = \underbrace{5 \oplus 3 \oplus 5}_{\text{unchanged}} \\ \downarrow \\ \text{change} \end{array} \quad \begin{array}{l} \text{first } \oplus 5 \oplus 3 \oplus 5 \\ \oplus \\ 0 \end{array}$$

Continue playing

Convince yourself that from a non-zero nim-sum, it is always possible to move to a zero nim-sum.



More nim!

Decide if the following games are N/P. If they are N, find all the winning moves.

1. Nim(1, 2, 4)
2. Nim(6, 7, 8, 9)
3. Nim(3, 4, 5, 5)