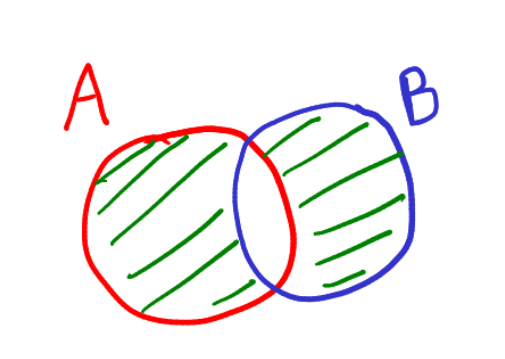
Games, graphs, and machines

Anand Deopurkar

July 22, 2025

The symmetric difference

Suppose A and B are represented by the circles below. Using the operations of union, intersection, and difference, express the shaded set.



Set operations

Is the following true or false: $|A - B| = |A| - |B|$. If it is true, explain why. If it is not true, give a counter-example.

Function or not?

Do the following rules define functions?

1. $f : \mathbb{Z} \to \mathbb{Z}$ defined by $f(s) = s^2$.
2. $f : \mathbb{Z} \to \mathbb{Z}$ defined by $f(s) = s/2$.
3. $f : \mathbb{Z} \to \mathbb{R}$ defined by $f(s) = s/2$.
4. $f : \mathbb{R} \to \mathbb{R}$ defined by $f(s) = \sqrt{s}$.
5. $f : \mathbb{R} \to \mathbb{R}$ defined by

$$f(s) = \begin{cases} 1 \text{ if } s > 0 \quad -1, \\ \text{ if } s < 0 \end{cases}.$$

Number of functions

Suppose $A = \{1, 2, 3\}$ and $B = \{1, 2, 3, 4\}$.

1. How many functions are there from A to B?

Number of functions

Suppose $A = \{1, 2, 3\}$ and $B = \{1, 2, 3, 4\}$.

1. How many functions are there from A to B?

2. How many of these are injective functions?

Number of functions

Suppose $A = \{1, 2, 3\}$ and $B = \{1, 2, 3, 4\}$.

1. How many functions are there from A to B?

2. How many of these are injective functions?

3. How many of these are surjective functions?

Number of functions (continued)

Suppose A has size n and B has size m.

How many functions are there from A to B?

Number of functions (continued)

Suppose A has size n and B has size m.

How many functions are there from A to B?

How many functions are injective?

Number of functions (continued)

Suppose A has size n and B has size m.

How many functions are there from A to B?

How many functions are injective?

How many functions are surjective?

Number of functions (continued)

Suppose A has size n and B has size m.

How many functions are there from A to B?

How many functions are injective?

How many functions are surjective?

`https://oeis.org/A019538`

The inverse function

Suppose $f : S \to T$ is a bijection.

The inverse of f is the function $g : T \to S$ defined by the property that if $t = f(s)$ then $s = g(t)$.

1. Suppose $f : \{1, 2, 3\} \to \{1, 2, 3\}$ sends $1 \mapsto 2, 2 \mapsto 3, 3 \mapsto 1$. Find its inverse g.

The inverse function

Suppose $f : S \to T$ is a bijection.

The inverse of f is the function $g : T \to S$ defined by the property that if $t = f(s)$ then $s = g(t)$.

1. Suppose $f : \{1, 2, 3\} \to \{1, 2, 3\}$ sends $1 \mapsto 2, 2 \mapsto 3, 3 \mapsto 1$. Find its inverse g.

2. Find a function $f : \{1, 2, 3\} \to \{1, 2, 3\}$ which is its own inverse.

The inverse function

Suppose $f : S \to T$ is a bijection.

The inverse of f is the function $g : T \to S$ defined by the property that if $t = f(s)$ then $s = g(t)$.

1. Suppose $f : \{1, 2, 3\} \to \{1, 2, 3\}$ sends $1 \mapsto 2, 2 \mapsto 3, 3 \mapsto 1$. Find its inverse g.

2. Find a function $f : \{1, 2, 3\} \to \{1, 2, 3\}$ which is its own inverse.

3. Find a function $f : \mathbb{R} \to \mathbb{R}$ which is its own inverse.

The inverse function

Suppose $f : S \to T$ is a bijection.

The inverse of f is the function $g : T \to S$ defined by the property that if $t = f(s)$ then $s = g(t)$.

1. Suppose $f : \{1, 2, 3\} \to \{1, 2, 3\}$ sends $1 \mapsto 2, 2 \mapsto 3, 3 \mapsto 1$. Find its inverse g.

2. Find a function $f : \{1, 2, 3\} \to \{1, 2, 3\}$ which is its own inverse.

3. Find a function $f : \mathbb{R} \to \mathbb{R}$ which is its own inverse.