WEEK 2 WORKSHOP

MATH2301, SEMESTER 2, 2025

1. Sets and set notation

1.1. **Problem.** Some of the following sets are the same, and some are different. Decide and discuss.

(1) $\varnothing$

(2) $\mathrm{Pow}(\varnothing)$

(3) $\varnothing \cap \mathrm{Pow}(\varnothing)$

(4) $\varnothing \cup \mathrm{Pow}(\varnothing)$

(5) $\varnothing \times \mathrm{Pow}(\varnothing)$

(6) $\mathrm{Pow}(\varnothing) \times \mathrm{Pow}(\varnothing)$

1.1.1. *Solution.* Here are all the sets in plainer language.

(1) The empty set, $\varnothing$.

(2) The set containing the empty set, $\{\varnothing\}$.

(3) The empty set, $\varnothing$.

(4) The set containing the empty set, $\{\varnothing\}$

(5) The empty set, $\varnothing$.

(6) The set $\{(\varnothing, \varnothing)\}$.

1.2. **Problem.** The symbol $\exists$ means "there exists". The symbol $\forall$ means "for all". The symbols $\mathbf{Q}$, $\mathbf{Z}$, and $\mathbf{R}$ denote the sets of rational numbers, integers, and reals respectively.

Consider the following set:

$$\left\{x \in \mathbf{Q} \mid \exists y \in \mathbf{Z} \text{ such that } x = \frac{y}{2} + 1\right\}.$$

Some of the following sets are the same as the set described above, and some are not. Decide and discuss.

(1) $\left\{x \in \mathbf{Z} \mid \exists y \in \mathbf{Z} \text{ such that } x = \frac{y}{2} + 1\right\}$.

(2) $\{x \in \mathbf{Q} \mid \exists y \in \mathbf{Z} \text{ such that } 2x = y\}$.

(3) $\left\{x \in \mathbf{Z} \mid \forall y \in \mathbf{Q} \text{ we have } x = \frac{y}{2} + 1\right\}$.

(4) $\left\{x \in \mathbf{R} \mid \exists y \in \mathbf{Z} \text{ such that } x = \frac{y}{2} + 1\right\}$.

Date: July 28, 2025.

1.2.1. *Solution.* Here are all the sets described in plainer language. The original set is the set of all x that can be described as half an integer, plus one. All integers have this form; for example, $3 = 4/2 + 1$. Additionally, all half-integers also have this form. For example, $5/2 = 3/2 + 1$. Nothing else has this form: we can conclude this by solving for y in the equation given to get

$$y = 2(x - 1)$$

where $x \in \mathbf{Q}$ and $y \in \mathbf{Z}$. Since y is an integer, the quantity $x - 1$ can have a denominator of either 1 or 2.

So the original set consists of the integers and the half-integers.

(1) This set simply describes all the integers, because all integers can be expressed in the form shown. It is not the same as the original set.

(2) This set describes all rational whose denominator is either 1 or 2. This is the same as the original set.

(3) This set is the empty set: it is never true for a rational x that $x = y/2 + 1$ *for all* $y \in \mathbf{Q}$.

(4) This is the same as the original set: if the given equation holds true, then x must already belong to the rationals, so the constraint is the same as the constraint of the original set.

2. Relations and graphs

2.1. **Problem.** Give an example of a set S and a binary relation $R \subset S \times S$ that has the listed properties.

(1) Symmetric but not reflexive.

(2) Transitive but not reflexive.

2.1.1. *Solution.*

(1) Symmetric but not reflexive: Many possible examples. For instance, $\{(0,1),(1,0)\}$ on the set $\{0,1\}$.

(2) Transitive but not reflexive: Many possible examples. For instance, $\{(0,1),(1,2),(0,2)\}$ on the set $\{0,1,2\}$.

2.2. **Problem.** For each kind of relation mentioned, discuss what special property the graph of the relation must satisfy.

(1) Reflexivity

(2) Symmetry

(3) Being the I/O relation of a function

(4) Transitivity

2.2.1. *Solution.*

(1) Reflexivity: Every node must have a self-loop.

(2) Symmetry: For every arrow (x,y), we also have an arrow (y,x).

(3) Being the I/O relation of a function: Every node has exactly one outgoing edge.

(4) Transitivity: if it is possible to go from a vertex a to a vertex b by following a sequence of edges (following the arrows), then there is an edge from a to b. (This is not completely obvious, but I will leave it for you to convince yourselves t