

# Games, graphs, and machines



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# Stars and cats

Alphabet  $\Sigma = \{0, 1\}$ .

Languages  $L = \{0\}$  and  $M = \{1, 11, 111, 1111, \dots\}$ .

1.  $LM =$
2.  $ML =$
3.  $L^* =$
4.  $M^* =$
5.  $L^*M =$

## REGULAR EXPRESSIONS

A regular expression is a pattern that describes a set of strings. Regular expressions are constructed analogously to arithmetic expressions, by using various operators to combine smaller expressions...

### Character Classes and Bracket Expressions

A bracket expression is a list of characters enclosed by [ and ]...

### Anchoring

The caret ^ and the dollar sign \$ are meta-characters...

### The Backslash Character and Special Expressions

The symbols \< and \> respectively match the empty string...

### Repetition

A regular expression may be followed by one of several repetition operators:

- ? The preceding item is optional and matched at most once.
- \* The preceding item will be matched zero or more times.
- + The preceding item will be matched one or more times.
- {n} The preceding item is matched exactly n times.
- {n,} The preceding item is matched n or more times.
- {,m} The preceding item is matched at most m times. This is a GNU extension.
- {n,m} The preceding item is matched at least n times, but not more than m times.

### Concatenation

Two regular expressions may be concatenated; ...

### Alternation

Two regular expressions may be joined by the infix operator |;...

# Our regexes

- $\emptyset$   
 $\epsilon$   
0  
1
- Concatenation  $ab$   
alternation  $a|b$   
star  $a^*$ .

# Regular expressions

Explicitly write the language described by the regex.

1.  $01^*$
2.  $(0|1)^*$
3.  $(01)^*$
4.  $00^*10^*0$

# Building regexes

Find regular expressions that describe the following languages.

1.  $\emptyset$
2.  $\{\epsilon\}$
3.  $\{0, 00, 000, \dots\}$
4.  $\{w \mid w \text{ starts with } 0 \text{ and ends with } 1\}$