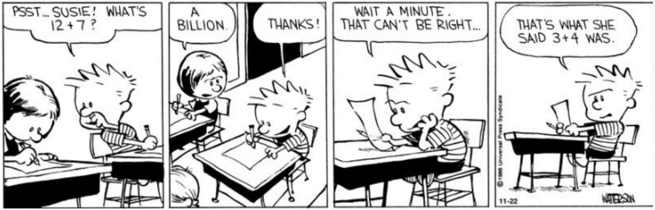
Games, graphs, and machines



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A much simplified arithmetic!

- $[0]$ = The set $\{0\}$.
- $[1]$ = The set of all positive numbers.

$$[0] + [0] = \qquad [0] \cdot [0] =$$

$$[0] + [1] = \qquad [0] \cdot [1] =$$

$$[1] + [1] = \qquad [1] \cdot [1] =$$

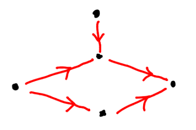
Boolean powers

Let $A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$. Find A^{*k} for $k = 1, 2, 3, \ldots$.

Can you explain the pattern using a graph?

Existence of paths

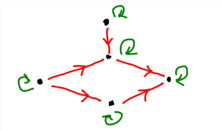
Let A be the adjacency matrix of the graph.



- Find A^{*k} for large k.
- Find $I + A + A^{*2} + \cdots + A^{*k}$ for large k.

Adding loops

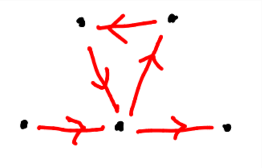
Suppose we add loops.



- Now what is A^{*k} for large k?
- What about $I + A + A^{*2} + \cdots + A^{*k}$?

Modular arithmetic?

Let A be the adjacency matrix of



Can you describe A^{*k} for all k?

Eventually periodic?

Let A be the boolean adjacency matrix of a graph.

Claim — The sequence of matrices $A, A^{*2}, A^{*3}, \ldots$ is eventually periodic.

True or false?