

WEEK 7 WORKSHOP
MATH2301, SEMESTER 2, 2025

In all problems, “binary string” refers to any string or word on the alphabet $\Sigma = \{0, 1\}$. Unless otherwise specified, our alphabet is always $\Sigma = \{0, 1\}$. If r is a regular expression, then $L(r)$ is the language described by r . This is the set of all strings that match r .

For this worksheet, make sure to discuss actively with your groupmates! Coming up with regexes that match what you want them to match, as well as describing the languages of given regexes, takes a lot of practice. It is a creative process and there is a lot of room for error. But you will learn the tricks of the trade quicker if you discuss frequently with others.

1. REGULAR EXPRESSIONS

1.1. **Problem.** If r is a regular expression, write down another regular expression s such that

$$L(s) = \{vwx \mid v, w, x \in L(r)\}.$$

1.2. **Problem.** Write down a regular expression whose language is

$$\{w \in \Sigma^* \mid w \text{ is any string except } 0 \text{ or } 1\}.$$

1.3. **Problem.** Write down a regular expression r that matches exactly those binary strings which (when thought of as numbers in base 2) are divisible by 8. (Let us assume that we only consider a binary string to represent a valid number if it either starts with a 1, or if the whole string equals 0.)

1.4. **Problem.** Let $r = 01^*0|10^*1$. Describe $L(r)$ in words.

1.5. **Problem.** Write down a regular expression for the language that contains exactly those strings without two consecutive 1s. Discuss and convince each other that you haven’t missed anything or have anything extra.

1.6. **Problem.** Write down a regular expression whose language is

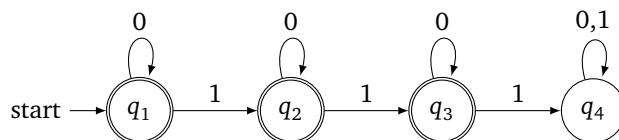
$$\{w \in \Sigma^* \mid w \text{ has exactly two } 0\text{s and at least two } 1\text{s}\}.$$

Discuss and convince each other that you haven’t missed anything or have anything extra.

1.7. **Problem.** For each of the problems 2, 3, and 4, try to come up with regular expressions that match precisely the strings that do not match the regular expression from the problem. You can either try to do this directly based on the descriptions of the language, or try to do it by manipulating the regular expressions. Can you find a systematic method for this?

2. DFAs

Answer the following questions about the DFA shown below.



2.1. **Problem.** What is the set of states?

2.2. **Problem.** What is the start state?

2.3. **Problem.** What is the set of accept states?

2.4. **Problem.** What is the transition function? Fill in the following table.

δ	0	1
q_1		
q_2		
q_3		
q_4		

2.5. **Problem.** Can you figure out a description for the language of this automaton?

2.6. **Problem.** Can you write down a regular expression that recognises the same language?