THE UNIVERSITY OF AUCKLAND

SEMESTER TWO 2019
Campus: City

COMPUTER SCIENCE

Algorithms and Data Structures

(Time allowed: Fifty minutes)

NOTE:

- Enter your full name and your student ID number on the top of the Teleform sheet.
- Sign your full name and write your ID number on the cover of this test script (see below).
- Keep this customized test script until your marks are finalized on Canvas.
  (On request, you may need to physically bring it to the course coordinator.)
- Shade in completely each answer on the Teleform sheet provided.
- All questions are worth the same number of marks.
- Use of any calculators or electronic devices is not permitted.

Student Signature: ________________________________

Student ID: ________________________________
1. Consider Bubble-sort algorithm below:
   
i) The algorithm splits a list of size $n$ into sorted (tail) and unsorted (head) sublists.
   
i) The sorted sublist is initially of size 0.
   
i) If the size of the unsorted list is 0 or 1, return the list.
   
i) Compare every two consecutive elements of the unsorted sublist and swap them if they are not in the correct order.
   
i) The last element of the unsorted sublist becomes the first element of the sorted sublist.
   
i) Go to iii)

\[\text{function BUBBLESORT(array } a[0..n-1], k)\]
\[\text{if } k < 1 \text{ then}\]
\[\text{return } a\]
\[\text{for } i \leftarrow 0 \text{ to } k - 1 \text{ do}\]
\[\text{if } a[i] > a[i+1] \text{ then}\]
\[\text{Swap } a[i] \text{ and } a[i+1]\]
\[\text{BubbleSort}(a,k-1)\]

An elementary operation in this question is a comparison or a swap of two elements of the list. What is the recurrence relation for this algorithm in the worst case?

A. $T(n) = T(n - 1) + 2(n - 1), T(0) = 0$
B. $T(n) = T(n - 3) + 2, T(0) = 7$
C. $T(n) = T(n - 1) + n \log_2(n), T(0) = 0$
D. $T(n) = T(n - 1) + n, T(0) = 0$
E. $T(n) = 2T(n - 1) + 1, T(0) = 0$

2. Selection sort is executed on array $[6,5,4,3,2,1]$. At certain step of the algorithm we have array $[1,2,3\text{, }4\text{, }5\text{, }6\text{ ]}$. Choose the array which will be produced at the next step of the algorithm.

\[\text{A. } [1,2,3\text{, }4\text{, }5\text{, }6\text{ ]}\]
\[\text{B. } [1,2,4\text{, }3\text{, }5\text{, }6\text{ ]}\]
\[\text{C. } [1,2,3,4,5,6\text{ ]}\]
\[\text{D. } [1,2,3\text{, }4\text{, }5\text{, }6\text{ ]}\]
E. All the other answers are incorrect
3. Consider Bubble-sort algorithm below:

i) The algorithm splits a list of size \( n \) into sorted (tail) and unsorted (head) sublists.

ii) The sorted sublist is initially of size 0.

iii) If the size of the unsorted list is 0 or 1, return the list.

iv) Compare every two consecutive elements of the unsorted sublist and swap them if they are not in the correct order.

v) The last element of the unsorted sublist becomes the first element of the sorted sublist.

vi) Go to iii)

```cpp
function BUBBLE_SORT(array a[0..n−1], k)
    if k < 1 then
        return a
    for i ← 0 to k − 1 do
        if a[i] > a[i + 1] then
            Swap a[i] and a[i + 1]
    BubbleSort(a, k-1)
```

An elementary operation in this question is a comparison or a swap of two elements of the list. What is the recurrence relation for this algorithm in average case?

A. \( T(n) = 2T(n - 1) + 1, T(0) = 0 \)
B. \( T(n) = T(n - 1) + \frac{3}{2}(n - 1), T(0) = 0 \)
C. \( T(n) = T(n - 1) + n \log_2(n), T(0) = 0 \)
D. \( T(n) = T(n - 1) + n, T(0) = 0 \)
E. \( T(n) = T(n - 3) + 2, T(0) = 7 \)

4. Which of the following statements about \( f(n) = 2^{n+2} + (n \log_{30}(n))^{10} + n^{100} + 6 \) is FALSE?

A. \( f(n) \) is \( \Omega(1) \)
B. \( f(n) \) is \( \Theta(n^{100}) \)
C. \( f(n) \) is \( \Omega(n^{101}) \)
D. \( f(n) \) is \( \Theta(2^n) \)
E. \( f(n) \) is \( O(n2^n) \)
5. We execute Quickselect to find the 4th order statistic of array [2, 6, 8, 13, 0, 4, 12, 5, 3, 1]. We select the first entry of the array as a pivot. Which subarray will we consider on the next step of the algorithm?

A. [13, 8, 4, 12, 5, 3, 6]
B. We do not need to consider any subarray, we have already found the 4th order statistic which is 2.
C. [2, 13, 8, 4, 12, 5, 3, 6]
D. [12, 5, 3, 1]
E. [2, 6, 8, 13]

6. An elementary operation in this question is only a comparison of two elements of an array. Find the number of elementary operations in the following piece of code below

```
function BLAH(array a[0..n-1])
    for i ← 0 to n-1 do
        for j ← 0 to n do
            if j is a power of 2 then
                Do constant C number of elementary operations
        return something
```

A. $nC([\log_2(n)] - 1)$
B. $nC([\log_2(n)] + 1)$
C. $nC([\log_2(n)] - 1)$
D. $nC[\log_2(n)]$
E. $(n-1)C([\log_2(n)] + 1)$

7. The running time for the following pseudocode fragment is $f(n)$. Which of the following statements is correct?

```
function BLAH(array a[0..n-1])
    j ← 1
    for i ← 0 to n-1 do
        while j ≤ n do
            Do constant C number of operations
            j ← 3j
    return something
```

A. $f(n)$ is $\Theta(n)$
B. $f(n)$ is $\Theta(n \log_3(n))$
C. $f(n)$ is $\Omega(n^2)$
D. All the other answers are incorrect
E. $f(n)$ is $O(\log_3(n))$
8. We want to sort array [7, 9, 3, 1, 4, 2, 10]. Which of the following statements corresponds to the quicksort algorithm execution?

A. Take 7 as a pivot. Partition the array and recursively sort two subarrays [4, 2, 3, 1] and [9, 10].
B. All the other choices are incorrect
C. Take 7 as a pivot. Partition the array and recursively sort two subarrays [4, 2, 3, 1] and [7, 9, 10].
D. Take 7 as a pivot. Partition the array and recursively sort two subarrays [2, 1, 3] and [4, 9, 10].
E. Take 7 as a pivot. Partition the array and recursively sort two subarrays [4, 2, 3, 1, 7] and [9, 10].

9. You are given a list which is almost sorted and there are only several elements which are in the wrong place. Which algorithm is the best in this situation?

A. insertion sort
B. heapsort
C. quicksort
D. mergesort
E. selectsort

10. Consider the binary search tree below. Which of the following orders can be an insertion order for this tree?

A. 10,5,15,1,11,7,17
B. 10,5,1,7,15,11,17
C. All given orders are insertion orders
D. All the other choices are incorrect
E. 10,15,5,17,7,11,1
11. Consider the binary heap below. Remove the highest priority node from it. Which array corresponds to the array representation of the resulting binary heap?

![Binary Heap Diagram]

A. [7, 9, 8, 3, 6, 5, 4, 2, 1, 10]
B. [9, 7, 8, 3, 6, 5, 4, 2, 1]
C. [9, 7, 8, 4, 6, 5, 2, 1, 3, 10]
D. [10, 9, 8, 7, 6, 5, 4, 2, 1]
E. [9, 7, 8, 3, 6, 5, 4, 2, 1, 10]

12. Which of the following statements about decision trees is TRUE?

A. A decision tree has $2^k + 1$ nodes at level $k$, for every $k$.
B. All the other choices are incorrect.
C. The height of a decision tree of an order on $n$ elements is $n \log(n)$.
D. To find the place of the element in a decision tree we need to compare it with all the other elements of the list.
E. The height of a decision tree for an order on $n$ elements is at least $\log(n!)$.

13. Given array $[1_a, 5, 1_b, 3, 4, 2]$, execute insertion sort on it. Which array can be seen on one of the steps of insertion sort?

A. $[1_a, 1_b, 4, 3, 5, 2]$ sorted unsorted
B. $[1_b, 1_a, 5, 3, 4, 2]$ sorted unsorted
C. $[1_a, 1_b, 3, 4, 5]$ unsorted sorted
D. $[1_a, 1_b, 5, 3, 4, 2]$ sorted unsorted
E. $[1_a, 1_b, 3, 4, 5]$ unsorted sorted
14. Which of the following arrays will give the worst case in the mergesort algorithm (count only comparisons of the list elements)?

A. None of the given arrays.
B. At least two of the other answers.
C. [5, 1, 7, 3]
D. [1, 2, 3, 4]
E. [8, 7, 6, 5]

15. Consider two functions \( f(n) = \sin(n) \) and \( g(n) = n^2 + 7 \). Which of the following statements is TRUE about these functions?

A. \( n^2 + 7 \) is \( O(\sin(n)) \)
B. \( \sin(n) \) is \( \Theta(n^2 + 7) \)
C. \( \sin(n) \) is \( O(n^2 + 7) \)
D. \( \sin(n) \) is \( \Omega(n^2 + 7) \)
E. All the other answers are incorrect

16. The recurrence relation \( T(n) = T(n - 1) + 2^n, T(0) = 1 \) has a solution which is in \( \Theta(f(n)) \). What is \( f(n) \)?

A. \( 2^n \)
B. \( \log_2(n) \)
C. \( n2^n \)
D. \( n^2 \)
E. \( n\log_2(n) \)