

QUIZ 5 (MATH2301, 2025)

Name: _____

UID: _____

Justifications are not required in any of the questions.

(1) (5 points) True or false?

(a) Let G be a weighted directed graph with 5 vertices. (Each weight is a non-negative number and all loops are included with weight 0.) If W is its weighted adjacency matrix, then the minimum cost to travel from vertex 2 to vertex 3 is the $(2, 3)$ entry of $W^{\odot 7}$.

True False .

(b) Let A be the boolean adjacency matrix of a directed graph. If n is large enough, then $A^{*n} = A^{*(n+1)} = A^{*(n+2)} = \dots$.

True False .

(c) Let W be a matrix. Let \oplus denote the min-plus addition. Then $W \oplus W = W$.

True False .

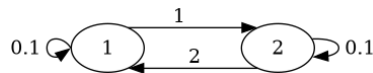
(d) Let A be a boolean matrix. There always exists a boolean matrix B such that $A + B$ is the zero matrix.

True False .

(e) Let A be a boolean matrix and let I be the identity matrix considered as a boolean matrix. Then $(I + A)^{*2} = I + A + A^{*2}$.

True False .

(2) (3 points) Let W be the weighted adjacency matrix of the following graph.



Find $W^{\odot 10}$.

(3) (2 points) Find $W \oplus W^{\odot 2} \oplus \dots \oplus W^{\odot 2025}$.

SOLUTIONS

- (1) (a) True.
(b) False (for example, take a 3-cycle $1 \rightarrow 2 \rightarrow 3 \rightarrow 1$).
(c) True.
(d) False (in fact, this almost never happens).
(e) True.

(2)

$$W^{\odot 10} = \begin{pmatrix} 1 & 1.9 \\ 2.9 & 1 \end{pmatrix}$$

(3)

$$W + \dots + W^{\odot 2025} = \begin{pmatrix} 0.1 & 1 \\ 2 & 0.1 \end{pmatrix}$$