

Games, graphs, and machines



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Grundy value = Nim sum for non-nim games

How to compute the Grundy value?

1. All sink states get 0.
2. Each state gets **mex** of its children.

Mex = “minimal excluded” (the smallest among the values not found among the children).

Example

The Grundy value of 2×2 Chomp.

Example

The Grundy value of Gobble starting with 10 berries and allowed to eat 1 or 2 berries.

Example

The Grundy value of $Nim(5)$.

Theorem

1. Grundy value of $G = 0$ if and only if G is a P -game.
2. Grundy value of $G \neq 0$ if and only if G is an N -game.
3. Grundy value of $G + H = \text{Grundy value of } G \oplus \text{Grundy value of } H$.

Application: how to play Chomp + Nim

Play (and win!) 2×2 -Chomp + Nim(4, 7)