ASSIGNMENT 3 (DUE 15 AUGUST 2025)

MATH2301, SEMESTER 2, 2025

1. General remarks

(1) The assignment is due on gradescope.

(2) Please read the academic integrity policy for assignments.

(3) The words "show" and "prove" are synonyms. You may not be used to writing formal mathematical proofs, which is OK. Write a justification in plain language that would convince the reader.

(4) If you are having trouble with any of the points, come and discuss with me in office hours. It is part of my job to help you understand this stuff, so please use my time!

2. Problems

2.1. **Problem.** The units digit of a perfect square can only be $0, 1, 4, 5, 6$, or 9.
What are the possible units digits of

(1) perfect cubes (of positive integers)?

(2) perfect fourth powers (of positive integers)?

Explain how you got your answers.

2.2. **Problem.** Let S be the divisor poset of 100. Suppose $f : S \to \mathbf{Z}$ is a rank function such that $f(1) = 0$.

(1) Find $f(4)$ and $f(10)$.

(2) Find all $d \in S$ such that $f(d) = 3$.

You should be able to justify your answers, but you need not submit any justification.

2.3. **Problem.** Let $S = \{(a, b) \in \mathbf{Z}^2 \mid a < b\}$. Define $\preceq$ on S by the rule

$$(a, b) \preceq (c, d) \text{ if } c \leqslant a \text{ and } b \leqslant d.$$

(Think of this as the interval $[a, b]$ is contained in the interval $[c, d]$).

(1) Recall the notion of a maximal chain from Wednesday's lecture. All maximal chains ending at $(0, 10)$ have the same length. What is this length? Our convention is that the length of a chain is the number of elements in the chain. So, for example, the chain $(1, 3) \preceq (0, 4) \preceq (0, 10)$ is of length 3.

(2) Every element of S has the same number of immediate successors. How many?

2.4. **Problem.** Let $S = \{1, 2, 3, 4\}$. Find all partial orders on S in which 1 is the minimum and 4 is maximal and $2 \preceq 3$. (Describe them by drawing their Hasse diagrams.)