Games, graphs, and machines

Relations

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Relations

A relation between A and B is a subset of $A \times B$.

The number of relations

Suppose $A = \{1, 2, 3\}$ and $B = \{1, 2, 3, 4\}$. How many relations are there between A and B?

Reflexive relations

We say that a relation $R \subset S \times S$ is *reflexive* if for all $s \in S$, we have $(s, s) \in S$.

Are the following relations reflexive?

1. $\leq$ on $\mathbb{R}$
2. $<$ on $\mathbb{R}$
3. the relation R on $\mathbb{Z}$ defined by $(a, b) \in \mathbb{Z}$ if 2 divides $a + b$.
4. Same as above, but with 2 replaced by 3.

Symmetric relations

We say that a relation $R \subset S \times S$ is *symmetric* if for all $s \in S$ and $t \in S$, if $(s, t) \in R$ then $(t, s) \in R$.

Find a relation on $\mathbb{Z}$ that is symmetric and one that is not symmetric.

Transitive relations

We say that a relation $R \subset S \times S$ is *transitive* if for all $a, b, c \in S$ if $(a, b) \in R$ and $(b, c) \in R$ then $(a, c) \in R$.

Find a relation on $\mathbb{R}$ that is transitive and one that is not transitive.

Transitive relations (continued)

Are the following relations transitive?

1. $\leq$ on $\mathbb{R}$
2. $<$ on $\mathbb{R}$
3. the relation R on $\mathbb{Z}$ defined by $(a, b) \in \mathbb{Z}$ if 2 divides $a + b$.
4. Same as above, but with 2 replaced by 3.

Input/Output relation

Consider $R \subset \mathbb{R} \times \mathbb{R}$ defined by

$$R = \{(x, y) \mid x^3 - xy + x - 1 = 0\}.$$

Is R the input/output relation of a function $f : \mathbb{R} \to \mathbb{R}$?