WEEK 3 WORKSHOP
MATH2301, SEMESTER 2, 2025

1. Modular arithmetic

1.1. **Problem.** Find all $x \in \mathbf{Z}/8\mathbf{Z}$ such that $[2]x + [4] = [0]$.

1.2. **Problem.** Find the last digit of 3^{101}.

Hint. This is equivalent to finding 3^{101} modulo 10. Calculate a few small powers, say $3^0, 3^1, 3^2, 3^4, \cdots$ modulo 10 and see if you can spot a pattern.

1.3. **Problem.** This is a somewhat open ended problem, so please attempt the other problems and come back to it. The quadratic formula says that the solutions to

$$ax^2 + bx + c = 0$$

are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Suppose $a, b, c \in \mathbf{Z}/n\mathbf{Z}$ and we are looking for x also in $\mathbf{Z}/n\mathbf{Z}$. Does the quadratic formula make sense? If it makes sense, is it correct?

2. The seven-dwarves poset

2.1. **Problem.** Consider the set of the names of the dwarfs from Snow White: {Bashful, Doc, Dopey, Grumpy, Happy, Sleepy, Sneezy}. Consider a partial order relation on this set, where $x \preceq y$ if the length of x is less than or equal to the length of y, and if x comes before y in alphabetical order. Draw the Hasse diagram of this partial order relation. Is it a total order?

2.2. **Problem.** Does the poset have maximum or minimum elements? Find all minimal and maximal elements.

3. The subset poset and the hypercube

Let $A = \{1, \cdots, n\}$ and let $S = \mathrm{Pow}(A)$. Instead of writing a subset $T \subset A$ by listing its elements, write it as an n-tuple of 0's and 1's so that the i-th place is 1 if $i \in T$ and 0 if $i \notin T$. For example, if $n = 3$, then the subset $\{1, 3\} \subset \{1, 2, 3\}$ is encoded by $(1, 0, 1)$.

3.1. **Problem.** Describe the subset relation in terms of the corresponding n-tuples. That is, if $T \subset T'$, what can you say about the n-tuples that encode T and T'?

3.2. **Problem.** Describe when an n-tuple is an immediate successor of another n-tuple under this relation.

3.3. **Problem.** Take $n = 3$. Draw the Hasse diagram in 3 dimensions by plotting the triples at the corresponding point in $\mathbf{R}^3$. So $(0, 0, 0)$ is at the origin and $(1, 1, 1)$ is at the point $(1, 1, 1)$. Draw the arrows representing immediate successors (we can not drop the arrowheads because there is no clear "up" or "down"). What shape do you get? As a warmup, do the exercise with $n = 2$ or even $n = 1$. What shape would you get in higher dimensions?

Date: August 4, 2025.