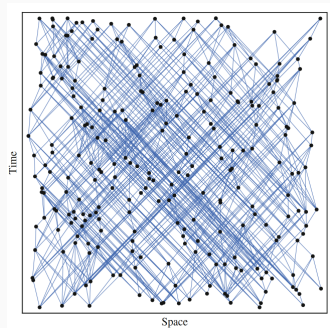


Games, graphs, and machines



August 5, 2025

Product poset

Let \leq be the usual order on \mathbb{R} . Define \preceq on $\mathbb{R} \times \mathbb{R}$ by

$$(a, b) \preceq (c, d) \text{ if } a \leq b \text{ and } c \leq d.$$

1. Give an example of two incomparable elements under \preceq .
2. Plot all elements that are $\preceq (2, 3)$.
3. Plot all elements (x, y) with $(1, 1) \preceq (x, y) \preceq (2, 3)$.

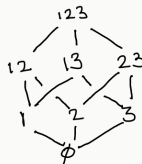
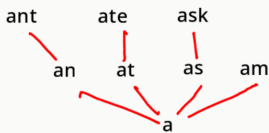
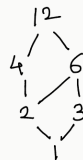
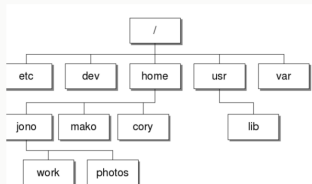
Divisor poset as product poset

Draw the Hasse diagrams of

1. the divisor poset of 12
2. the product poset of $\{1, 2, 3\}^{\leq}$ and $\{1, 2\}^{\leq}$.

Max/min

In all the examples so far, identify the maximum (if it exists), the minimum (if it exists), all maximal elements, all minimal elements.



Product poset \mathbb{R}^2

Immediate successors

Let S be the divisor poset of 60. What are the immediate successors of 3?

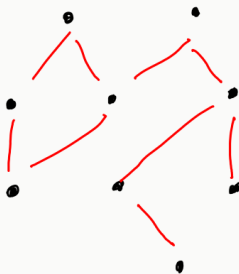
Immediate successors

Let S be the poset of words with \preceq given by prefix.

- What are the immediate successors of “ant”?
- What is an element that succeeds “ant” but is not an immediate successor?

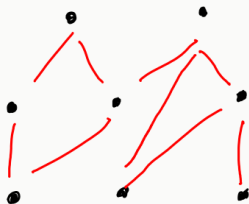
Rank function

Find a rank function on the following poset.



Rank function?

Find a rank function on the following poset.



When does a rank function exist?

Under what conditions does a rank function exist on a poset?