

WEEK 11 WORKSHOP
MATH2301, SEMESTER 2, 2025

1. NIM SUM

1.1. **Problem.** Find the nim sum of the following nim games and using it, find all winning (first) moves.

- (1) Nim(2, 3, 4)
- (2) Nim(2, 9, 10)

1.2. **Problem.** Construct 5-pile nim games that has one/three/five possible winning first moves. Is there a nim game that has two possible winning first moves? Why?

1.3. **Problem.** Consider Nim(4, 5, 6), which has nim sum $100 \oplus 101 \oplus 110 = 100$, which is 4. Find moves that take it to a state with nim sum 0, 1, 2, 3. Is there a move that takes it to a state with nim sum 4?

Repeat the procedure with another nim, say Nim(3, 3, 5). It has nim sum 5. Find moves that take it nim sum 0, 1, 2, 3, 4. Is there a move that takes it to a state with nim sum 5?

1.4. **Problem.** Convince your friends that the following is true: if a nim game has nim sum n , then it is possible to take it to a game with nim sum 0, 1, 2, ..., $n - 1$, but not n .

(If you are stuck, do the other problems and then come back to this one.)

2. DIVISOR CHOMP

The game of divisor chomp is played as follows. We start with all positive divisors of a number N written on the board (except 1). On their turn, a player erases a number d together with all d' such that d divides d' . For example, for $N = 12$, we will start with 2, 3, 4, 6, 12 on the board. If the first player chooses $d = 6$, then 6 and 12 are erased. Then, if the second player chooses $d = 3$, then only 3 is erased. And so on. The player who erases the last number wins.

2.1. **Problem.** Play divisor chomp with a few values of N .

2.2. **Problem.** Divisor chomp with $N = 200$ is actually 2×3 chomp in disguise. Why? Can you similarly interpret divisor chomp with $N = 900$?

3. POSET CHOMP

The game of poset chomp is played as follows. We start with a poset drawn on the board. On their turn, a player erases an element d together with all d' such that $d \leq d'$. The player who erases the last element of the poset wins.

3.1. **Problem.** The usual chomp is an example of poset chomp for a particular poset. Which poset?

3.2. **Problem.** These are the Hasse diagrams of some posets. Suppose we play poset chomp starting with those posets. Label the starting state as N or P .



(1)



(2)



(3)