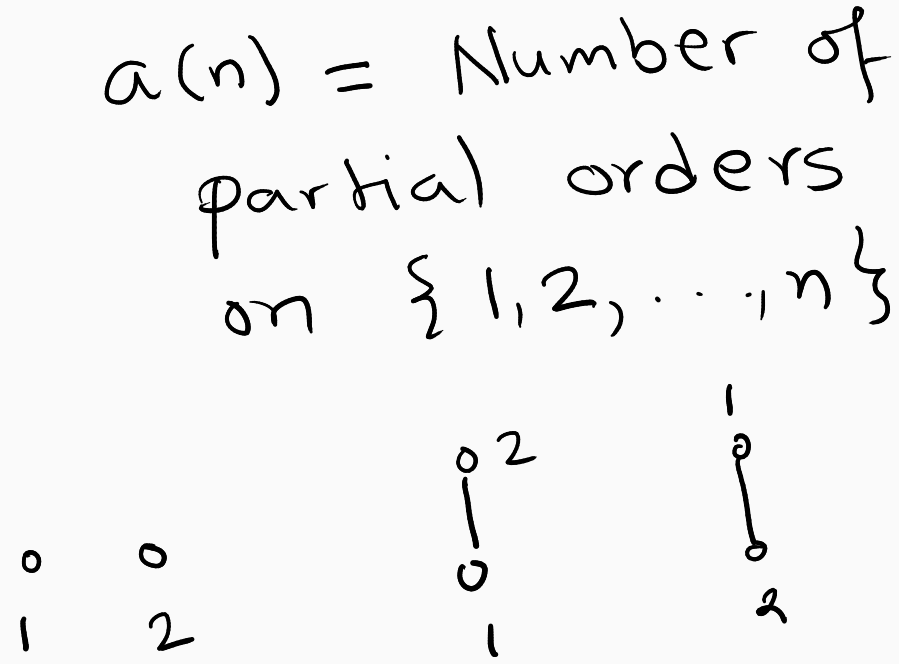
Games, graphs, and machines

n	a(n)
0	1
1	1
2	3
3	19
4	219
5	4231
6	130023
7	6129859
8	431723379
9	44511042511
10	6611065248783
11	1396281677105899
12	414864951055853499
13	171850728381587059351
14	98484324257128207032183
15	77567171020440688353049939
16	83480529785490157813844256579
17	122152541250295322862941281269151
18	241939392597201176602897820148085023



Chains

A *chain* in a poset is a sequence of distinct elements $a_1, \ldots, a_n$ such that

$$a_1 \preceq a_2 \preceq \cdots \preceq a_n.$$

The number n is the *length* of the chain.

Find a chain of length 3 in the subset poset of $\{1, 2, 3, 4\}$.

$\emptyset \preceq \{1\} \preceq \{1, 2\}$

$\{1\} \preceq \{1, 2, 3\} \preceq \{1, 2, 3, 4\}$

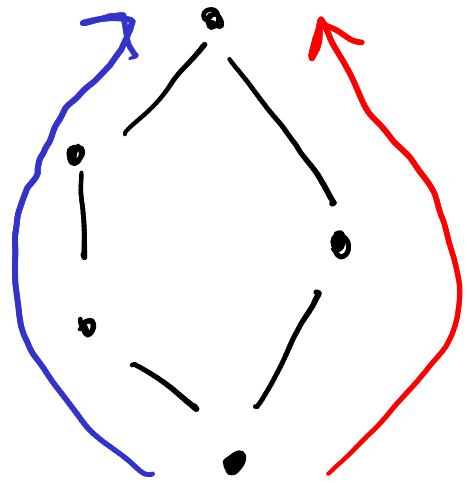
Maximal chains

- What could be the meaning of a *maximal chain*?
- Find a maximal chain in the subset poset of $\{1, 2, 3, 4\}$.

@ Chain of largest length

(b) a chain that cannot be made longer by extending it on either end or in the middle

maximal



maximal !

Maximal chains

- In the subset poset of $\{1, \ldots, n\}$, all maximal chains have the same length. True/false? $(n+1)$.
- In any poset, all maximal chains have the same length. True/false?

Counterexample: $n = 8$

Chain 1: $\{1\}$ $\{1,2\}$ $\{1,2,4\}$ $\{1,2,4,8\}$

Chain 2: $\{3\}$ $\{5\}$ $\{6\}$

goes up in size by 1

$\emptyset$ — — — — —

$\{1, 2, 3, \ldots, n\}$

$(n+1)$ elts.

Maximal chains and ranks

Let S be a finite poset.

Theorem ?

If all maximal chains in S have the length, then there exists a rank function on S.

test

- Verify the theorem for the subset poset of $\{1, \ldots, n\}$.
- Do you think the theorem is true? Why?

Satisfies the test.

thm say ∃ rk fun.

$f(S) = |S|$.

Converse?

Theorem
If all maximal chains in S have the length, then there exists a rank function on S.

The *converse* of "If A then B" is "If B then A".

Do you think the converse of the theorem is true?