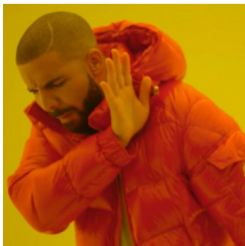


Games, graphs, and machines

Modular arithmetic

July 29, 2025

Visualising modular arithmetic

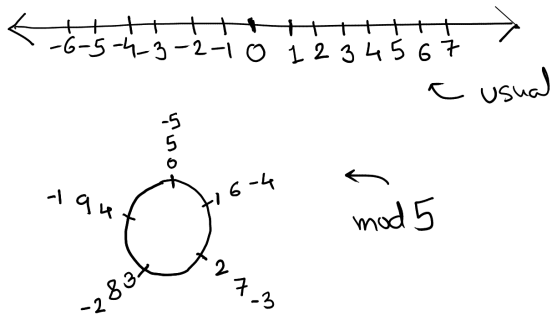


Number
line



Number
circle

Visualising modular arithmetic



Modular operations + and \times

To find $A + B$ —

1. Pick $a \in A$ and $b \in B$.
2. Compute $a + b$.
3. The answer is the resulting equivalence class.

To find $A \times B$ — Same thing.

Key point Consistency

The answer does not depend on which a or b was chosen.

Laws of arithmetic

Both $+$ and \times satisfy the usual laws (commutativity, associativity, distributivity).

For example,

1. What is the negative of $\bar{3}$ modulo 7?
2. Compute $\bar{3} \times \bar{5} - \bar{13} \pmod{8}$.

Laws of arithmetic: surprises

Some things are different! For example, it may happen that $a \times b = 0$ but $a \neq 0$ and $b \neq 0$.

Let $S = \mathbb{Z}/12\mathbb{Z}$. Find $A, B \in S$ such that $A \neq \bar{0}$ and $B \neq \bar{0}$ but

$$A \cdot B = 0.$$

Laws of arithmetic: surprises

What are the square-roots of $\bar{1}$ in $\mathbb{Z}/8\mathbb{Z}$?

Modular equations

1. Solve $2x + 7 = 0$ in $\mathbb{Z}/5\mathbb{Z}$.

2. Solve $2x + 7 = 0$ in $\mathbb{Z}/6\mathbb{Z}$.