

Games, graphs, and machines



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Stars and cats

Alphabet $\Sigma = \{0, 1\}$. Strings : $\epsilon, 0, 1, 00, 01, \dots$

Languages $L = \{0\}$ and $M = \{1, 11, 111, 1111, \dots\}$.

1. $LM = \{01, 011, 0111, \dots\}$

2. $ML = \{10, 110, 1110, \dots\}$ $L^* = \{\epsilon\} \cup L \cup LL \cup LLL \cup \dots$

3. $L^* = \{\epsilon, 0, 00, 000, \dots\}$

4. $M^* = \{\epsilon, 1, 11, 111, \dots\} = \bigcup_n \underbrace{LLL \dots L}_{n \text{ times}}$


5. $L^*M = \{1, 11, 01, 111, 011, 001, \dots\}$

$\left\{ \begin{array}{l} \text{a block of zeros} \\ \text{of any len} \end{array} \right. \cdot \left\{ \begin{array}{l} \text{a block of ones of len} \geq 1 \end{array} \right\}$

6. $\{\epsilon\} \cdot \{\epsilon\} = \phi$
 $\phi^* = \{\epsilon\}$



REGULAR EXPRESSIONS

A regular expression is a pattern that describes a set of strings. 
Regular expressions are constructed analogously to arithmetic expressions, by using various operators to combine smaller expressions...

Character Classes and Bracket Expressions

A bracket expression is a list of characters enclosed by [and]...

Anchoring

The caret ^ and the dollar sign \$ are meta-characters...

The Backslash Character and Special Expressions

The symbols \< and \> respectively match the empty string...

Repetition

A regular expression may be followed by one of several repetition operators:

- ? The preceding item is optional and matched at most once.
- * The preceding item will be matched zero or more times.
- + The preceding item will be matched one or more times.
- {n} The preceding item is matched exactly n times.
- {n,} The preceding item is matched n or more times.
- {,m} The preceding item is matched at most m times. This is a GNU extension.
- {n,m} The preceding item is matched at least n times, but not more than m times.

Concatenation

Two regular expressions may be concatenated; ...

Alternation

Two regular expressions may be joined by the infix operator |;...

atomic
+
rules of
combinations

Our regexes

- \emptyset \longrightarrow No string matches
- ϵ \longrightarrow $\{\epsilon\}$
- 0 \longrightarrow $\{\epsilon\}$
- 1 \longrightarrow $\{0\}$
- Concatenation ab
- alternation $a|b$ \longrightarrow $\{1\}$
- star a^* \longrightarrow "OR"

Regular expressions

Explicitly write the language described by the regex.

1. 01^* = $\{0, 01, 011, 0111, 01111, \dots\}$ \Leftarrow String that match

2. $(0|1)^*$ = $\{0, 1\}^*$ = $\{\epsilon, 0, 1, \dots\}$

3. $(01)^*$ = $\{\epsilon, 01, 0101, 010101, \dots\}$ \rightarrow all strings.

4. 00^*10^*0 = $\{\epsilon, 01, 0101, 010101, \dots\}$ \rightarrow all strings.

$\{ \underbrace{00\dots0}_{\geq 0} 1 \underbrace{0\dots0}_{\geq 0} 0 = \underbrace{0\dots0}_{\geq 1} 1 \underbrace{0\dots0}_{\geq 1} \}$
 $0^m 1 0^n \quad m, n \geq 1$

Order of ops

*
concat
OR

$\{\epsilon, 0, 1, 00, 11, 000, 111, \dots\}$

$0^* | 1^*$

What if we want to capture

010

00100

0001000

⋮

$\{0^n 1 0^n\}$

Spoiler

Can Not be done
using a regex!

Building regexes

Find regular expressions that describe the following languages.

1. \emptyset \emptyset
2. $\{\epsilon\}$ $\epsilon, \emptyset^+, \epsilon^+$
3. $\{0, 00, 000, \dots\}$ $00^+, 0^+0$
4. $\{w \mid w \text{ starts with } 0 \text{ and ends with } 1\}$

$0 (0|1)^+ 1$
↑ ↑ ↑
 anything.