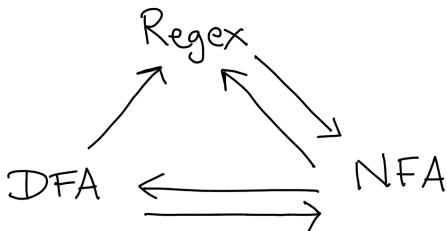


Games, graphs, and machines



October 1, 2025

All are equivalent



Regular language

Language?

Language \rightarrow Regex /
DFA/
NFA

Sometimes impossible!

Language ~~→~~ Regex /
DFA /
NFA

Pumping Lemma

Myhill-Nerode thm

Characterises
regular languages

Automatons are very limited

Even “easy” patterns are beyond automatons!

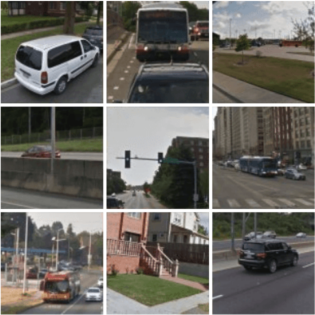
$0^m 1^m$

Palindrome

more 0's than 1's.

What about...

Select all images with a
bus
Click verify once there are none left.



⌂ 🎧 ⓘ

VERIFY

Using machine learning to identify undiagnosable cancers

A new model that maps developmental pathways to tumor cells may unlock the identity of cancers of unknown primary.

Turing machines

Turing machine = Finite automaton + memory

external

Can recognise :

$0^m 1^m$

palindromes

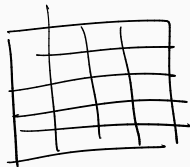
squares

powers of 2

⋮

Turing machines?

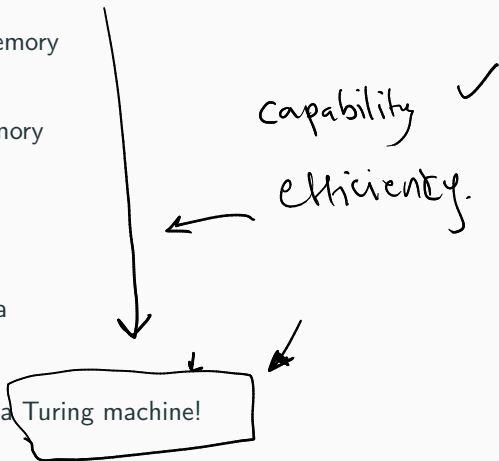
- Two dimensional memory
- Many reading heads
- Random-access memory
- Non-determinism
- Parallelism
- Cellular automata
- Crystalline automata
- ...



Turing machines?

- Two dimensional memory
- Many reading heads
- Random-access memory
- Non-determinism
- Parallelism
- Cellular automata
- Crystalline automata
- ...

... are all equivalent to a Turing machine!



Church-Turing thesis

Anything that is computable is computable by a Turing machine.

Non-computable patterns?

① Truth vs falsehoods (Gödel's incompleteness)

② Correct vs incorrect computer programs (Halting problems)

↳ Input : computer program.

Output : Accept if program halts
Reject if ——— doesn't halt.

Thm : (Halting problem - Turing)

There is no Turing machine that can do the halting problem.

1950's

Further developments

How efficiently computable?

- Polynomial time versus exponential time?

"easy"

"hard"

↙
sensitive to the model
of computation.

Further developments

How efficiently computable?

- Polynomial time versus exponential time?
- Multiplying $n \times n$ matrices: best method takes about $n^{2.37}$ operations. Can we do faster?

$$n^3$$

$$n^{2.37}$$

Further developments

How efficiently computable?

- Polynomial time versus exponential time?
- Multiplying $n \times n$ matrices: best method takes about $n^{2.37}$ operations. Can we do faster?
- Can we do faster with parallelisation? Quantum computers? Probabilistic computation?

Futher questions

- Are there any physical processes that are more capable than a Turing machine?
- Is the human brain (theoretically) more capable?