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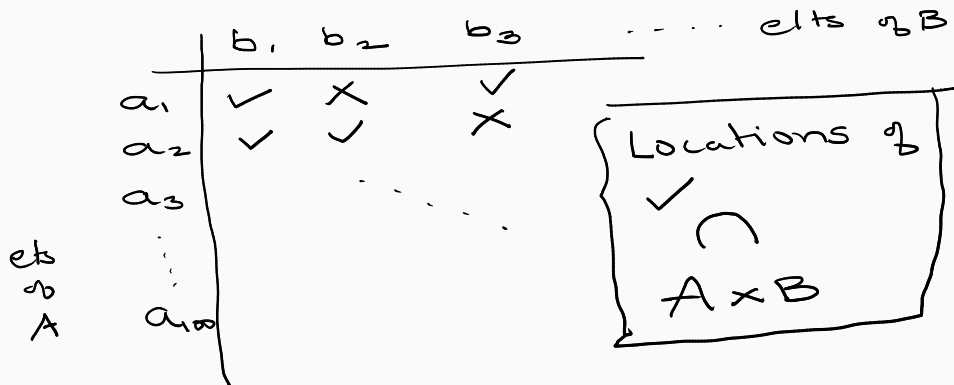
Games, graphs, and machines

Relations

July 23, 2025

Relations

A relation between A and B is a subset of $A \times B$.



Relations

A relation between A and B is a subset of $A \times B$.

$$R \subset A \times B$$

$A = \text{ANU Students.}$

$B = \{2025, 2024, \dots, 2000\}$

$(\text{Student}, \text{year}) \in R$

if student was in ANU in year.

The number of relations

Suppose $A = \{1, 2, 3\}$ and $B = \{1, 2, 3, 4\}$. How many relations are there between A and B ?

12

125

4096

Subset of
 $A \times B$

\Rightarrow

2^{12}

subsets of

a 12 elts set

2^6

2^6

$2^3 \cdot 2^3$

$64 \cdot 25$
 64

 4096

Reflexive relations

We say that a relation $R \subset S \times S$ is *reflexive* if for all $s \in S$, we have $(s, s) \in R$.

Are the following relations reflexive?

- ✓ 1. \leq on \mathbb{R} ($\{(x, y) \mid x \leq y\} \subset \mathbb{R}^2$)
- ✗ 2. $<$ on \mathbb{R}
- ✓ 3. the relation R on \mathbb{Z} defined by $(a, b) \in \mathbb{Z}^2$ if 2 divides $a + b$.
- ✗ 4. Same as above, but with 2 replaced by 3.

	s_1	s_2	...
s_1	✓		
s_2		✓	
...			✓

$$A = \{1, 2, 3\}$$

$$2^6$$

	1	2	3
1		✓	
2			✓
3			✓

How many refl. rel on A ?

6 remaining can be
✓, ✗

Symmetric relations

We say that a relation $R \subset S \times S$ is *symmetric* if for all $s \in S$ and $t \in S$, if $(s, t) \in R$ then $(t, s) \in R$.

Find a relation on \mathbb{Z} that is symmetric and one that is not symmetric.

Ex. symmetric

- ① $(a, b) \in R$ if 7 divides $a - b$
- ② $-||-$ if 7 divides $a + b$
- ③ \emptyset

Non-symmetric

- ① $(a, b) \in R$ if a divides b
- ② $(a, b) \in R$ if a is a power of 2 .

Symmetric relations

We say that a relation $R \subset S \times S$ is *symmetric* if for all $s \in S$ and $t \in S$, if $(s, t) \in R$ then $(t, s) \in R$.

~~Find a relation on \mathbb{Z} that is symmetric and one that is not symmetric.~~

$$A = \{1, 2, 3\}$$

How many symmetric relations?

Also $2^6 =$

	1	2	3
1	?	□	○
2	■	?	△
3	●	▲	?

Transitive relations

We say that a relation $R \subset S \times S$ is *transitive* if for all $a, b, c \in S$ if $(a, b) \in R$ and $(b, c) \in R$ then $(a, c) \in R$.

Find a relation on \mathbb{R} that is transitive and one that is not transitive.

Example

0. \emptyset

1. $(a, b) \in R$ if a is a power of 2.

2. \leq inequality

3. $<$ strict inequality

4. $=$ equality

Non-example

1. $(a, b) \in R$ if ab is an even integer.

$a=5$ $b=2$ $c=3$

2. \neq inequality

Transitive relations

We say that a relation $R \subset S \times S$ is *transitive* if for all $a, b, c \in S$ if $(a, b) \in R$ and $(b, c) \in R$ then $(a, c) \in R$.

~~Find a relation on \mathbb{R} that is transitive and one that is not transitive.~~

$$A = \{1, 2, 3\}$$

How many transitive
relations on an
n elt set.

Probably
an OEIS

↓
???

online encyclopedia of integer seq.

Transitive relations (continued)

Are the following relations transitive?

- ✓ 1. \leq on \mathbb{R}
- ✓ 2. $<$ on \mathbb{R}
- ✓ 3. the relation R on \mathbb{Z} defined by $(a, b) \in \mathbb{Z}$ if 2 divides $a + b$.
- ✗ 4. Same as above, but with 2 replaced by 3.

$$(2, 1) \in R \quad (1, 2) \in R$$

$$(2, 2) \notin R.$$

Input/Output relation

Consider $R \subset \mathbb{R} \times \mathbb{R}$ defined by

$$R = \{(x, y) \mid x^3 - xy + x - 1 = 0\}.$$

Is R the input/output relation of a function $f: \mathbb{R} \rightarrow \mathbb{R}$?