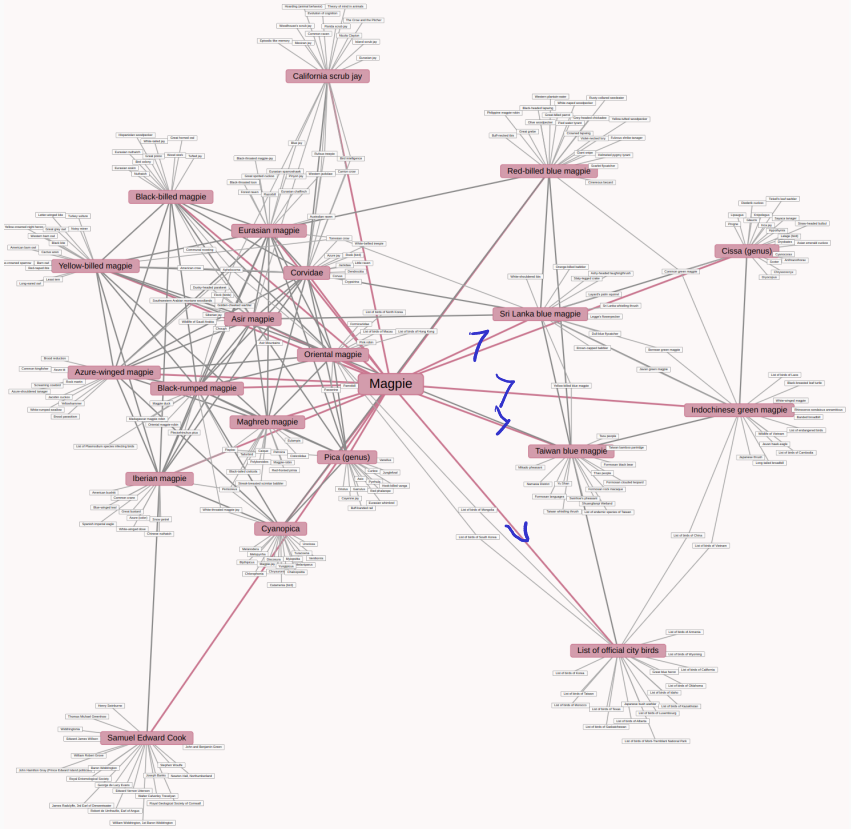
Games, graphs, and machines



August 11, 2025

Neighbour graph

undirected ⇒ adj matrix is symmetric.

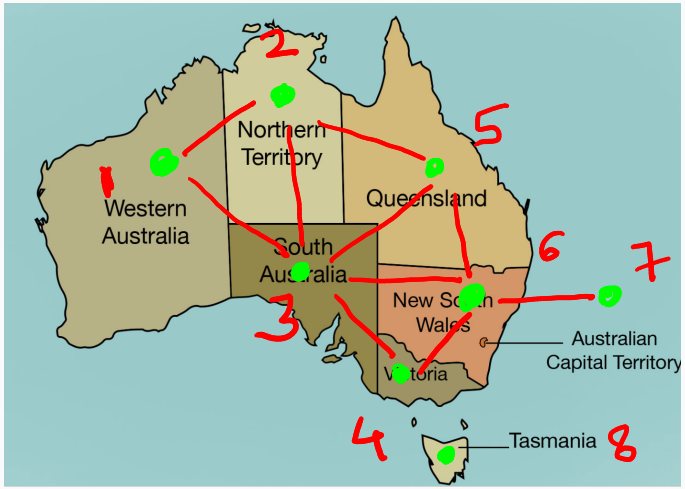
Draw the graph whose

- vertices are the states or territories of Australia,
- two vertices are joined by an edge if they share a border.

incidence matrix

Vertices →

↓ edges

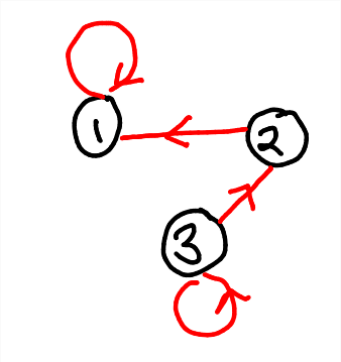


	1	2	3	4	5	6	7	8
1	0	1	1	0	0	0	0	0
2	1							
3	1							
4	0							
5	0							
6	0							
7	0							
8	0							

Write the adjacency matrix.

Another adjacency matrix

Write the adjacency matrix of the following directed graph.



(i, j) row i, col j = 1 if $i \to j$

	1	2	3
1	1	0	0
2	1	0	0
3	0	1	1

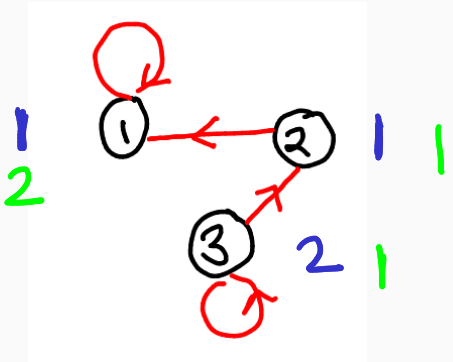
Degree of a vertex

- The *out-degree* of a vertex is the number of edges going out of it.
- The *in-degree* of a vertex is the number of edges coming into it.

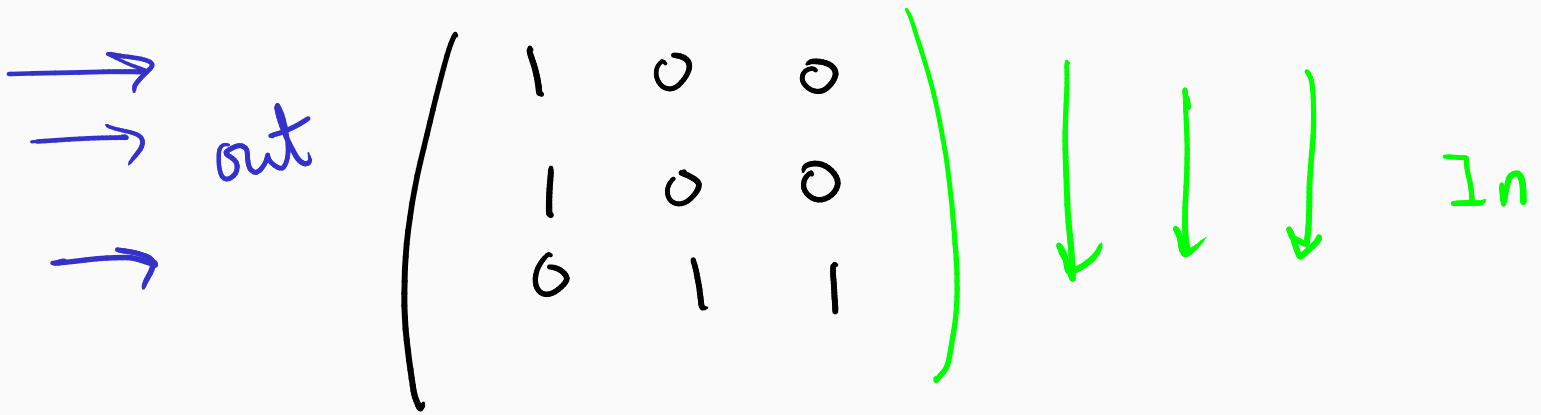
1. Find the incoming and outgoing degrees in the previous graph.

Out

In



2. How do you read off the degrees from the adjacency matrix?



In = out?

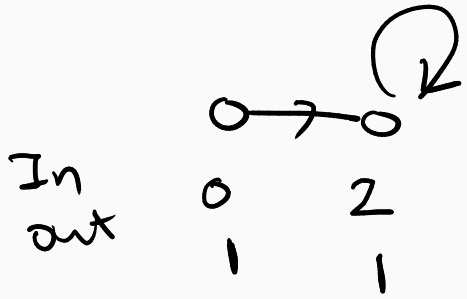
True or false: Directed

Sum of all in-degrees = Sum of out-degrees.

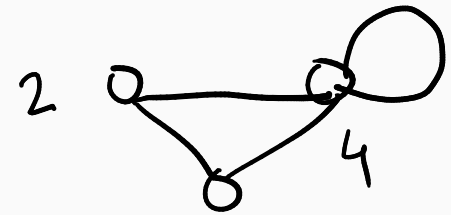
Thm :

||

Number of edges



Undirected :



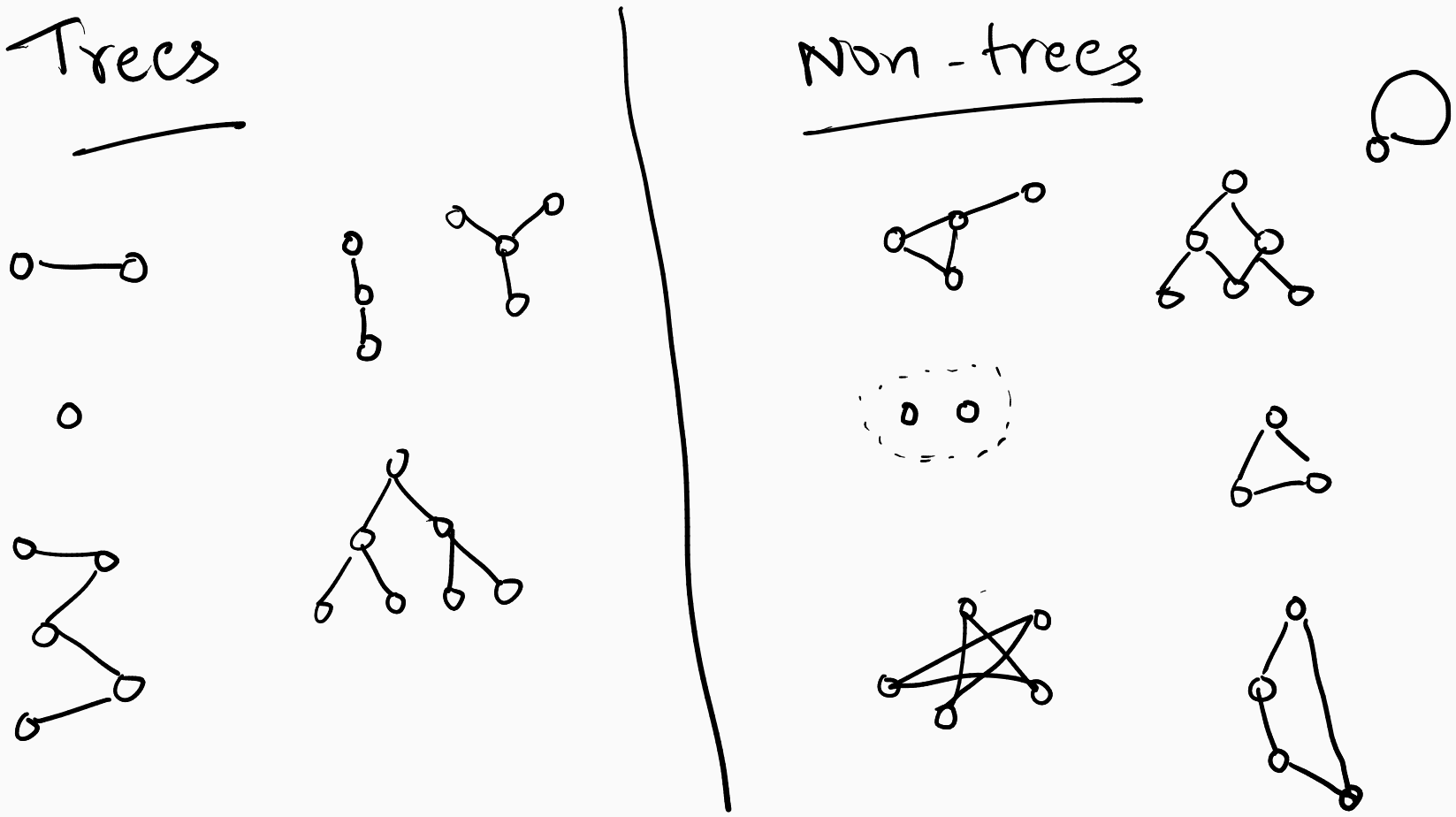
Thm : Sum of deg = 2 · # edges

↓

undirected "Handshake lemma"

Trees

An undirected graph is a *tree* if it is connected and has no cycles. Draw 3 examples of trees and 3 examples on non-trees.



Leaves

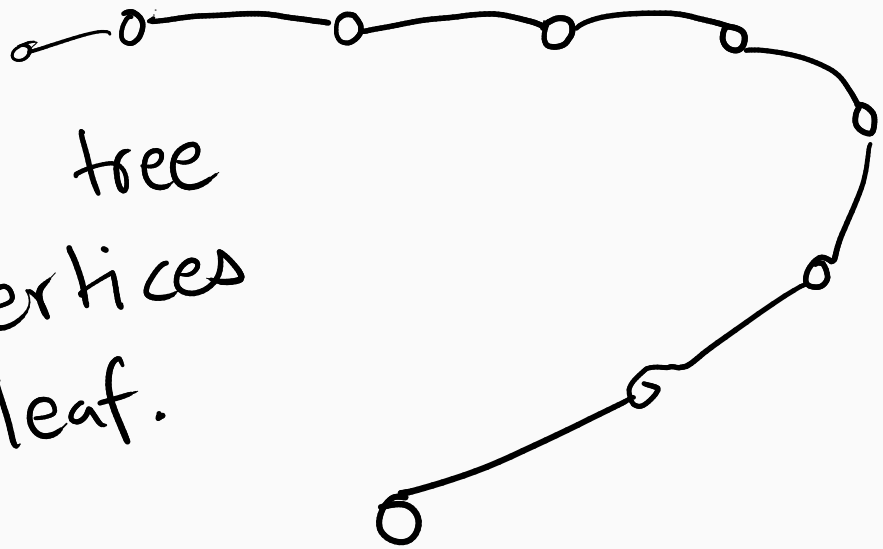
A *leaf* ~~of a tree~~ is a vertex of degree 1. (undirected)

True ~~or false~~: every tree has at least one leaf. , except o

finite ① ② every graph has at least one leaf; except o

① False: o ② Same counter-ex.

① True:



Every finite tree on ≥ 2 vertices has a leaf.

Edges of a tree

Count the number of vertices and edges in your trees.

- Can you see a pattern?
- Can you explain the pattern?