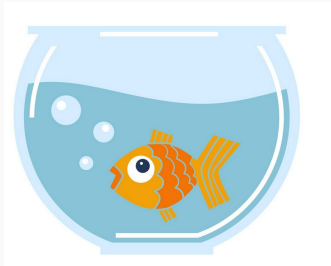


Games, graphs, and machines



September 24, 2025

Distinguishable strings

Let L be a language.

Say that L *distinguishes* x and y if there exists a z such that exactly one of xz or yz is in L .

L	L^c	
x	y	✓
xz	yz	✓
yz	xz	✓

Distinguishable strings

Let L be a language.

Say that L *distinguishes* x and y if there exists a z such that exactly one of xz or yz is in L .

Example

Let $L = 01^*0 \mid 10^*1$.

1. Does L distinguish 0 and 1?

2. What about 01 and 10?

3. What about 010 and 101?

x y

001 (101)

xz yz

yes : Produce $z = 01$

011 (101)

010, 101 $\in L$

010 $\not\in L$

101 $\not\in L$

Indistinguishable strings

Say that $x \sim_L y$ if L cannot distinguish x and y .

Proposition: \sim_L is an equivalence relation.

✓ ① Reflexive

✓ ② symmetric

③ transitive

$u \sim v$

$v \sim w$

\Rightarrow

$u \sim w$
✓

① u

vz

both in
or
both out

vz

② wz

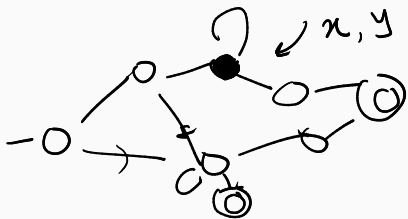
both in
both out

for all z
 $xz \ yz \in L$
OR $xz \ yz \notin L$

How do we know $x \sim y$?

Suppose L has a DFA M .

Proposition: If x and y end at the same state in M , then $x \sim_L y$.



Imagine
M reading xz
 yz

↓

M will reach the
same state!

How do we know $x \sim y$?

Suppose L has a DFA M .

Proposition: If x and y end at the same state in M , then $x \sim_L y$.

Proposition: The number of \sim_L equivalence classes is at most the number of states of M .

5 states

6 inequ. strings
 x_1, \dots, x_6

Example

Let $L = \{\text{Palindromes}\}$.

RACECAR

minim

Proposition: 01, 001, 0001, 00001, \dots are distinguishable.

00 11 01010 1011101 ✓

Aman a planacanal panama

010 0010 $Z = 0$

0000|0000
00000000

00000000|0000
00000000

Example

Let $L = \{\text{Palindromes}\}$.

Proposition: 01, 001, 0001, 00001, \dots are distinguishable.

Consequence: There is no DFA for L .

no NFA for L .
no regex for L .

The Myhill-Nerode Theorem

Theorem: L is regular if and only if \sim_L has finitely many equivalence classes.



recognisable by a DFA.