

QUIZ 4 (MATH2301, 2025)

Name: _____

UID: _____

Justifications are not required in any of the questions.

(1) (4 points) Select true or false. In all of the following, G is a directed graph and A is its adjacency matrix.

(a) The (i, j) entry of A^k is the number of paths of length at most k from vertex i to vertex j .

True

False

(b) If $A^n \neq 0$ for all n , then G must have a directed cycle.

True

False

(c) Suppose G has 4 vertices. If $A^5 = 0$, then we must have $A^4 = 0$.

True

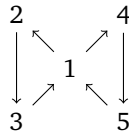
False

(d) If $A_{1,1}^2 \neq 0$ and $A_{1,1}^3 \neq 0$, then $A_{1,1}^n \neq 0$ for all $n \geq 2$.

True

False

(2) (2+2 points) Let G be the bowtie graph and let A be its adjacency matrix.



(a) Find $A_{1,1}^{24}$.

$A_{1,1}^{24} =$ _____

(b) Write one (i, j) for which $A_{i,j}^{26}$ is zero and one for which it is non-zero.

$A_{i,j}^{26} = 0$ for $(i, j) =$ _____ $A_{i,j}^{26} \neq 0$ for $(i, j) =$ _____

(3) (2 points) Draw a connected directed graph on at least 3 vertices for which $A^3 = 0$ but $A^2 \neq 0$.

SOLUTIONS

- (1) (a) False. It is the number of paths of length exactly k .
- (b) True. A long enough path must include a cycle.
- (c) True. The only way $A^4 \neq 0$ is if there is a cycle. And in that case, $A^5 \neq 0$ as well.
- (d) True. Paths of length 2 and 3 from 1 to 1 give paths of length $2 + 2, 2 + 3, 3 + 3, 2 + 2 + 3$, etc.
- (2) (a) The only way to go from 1 to 1 is by following the left half of the bowtie or the right half of the bowtie, both of which are of length 3. In path of length 24, there will be 8 iterations of left/right bowtie, and each can be chosen independently leading to $2^8 = 256$ possibilities.
- (b) $A_{i,j}^{26} = 0$ for $(i, j) = (1, 1)$, for example and $A_{i,j}^{26} \neq 0$ for $(i, j) = (1, 3)$. Lengths of paths from 1 to 1 are always divisible by 3; lengths of paths from 1 to 2 are $1 \pmod{3}$; from 1 to 3 are $2 \pmod{3}$, etc.
- (3) Many possibilities, for example: $1 \rightarrow 2 \rightarrow 3$.