Games, graphs, and machines

October 15, 2025

Rules of Kayles

1. Knock down a single pin or two adjacent pins.
2. The player to knock down the last pin wins.

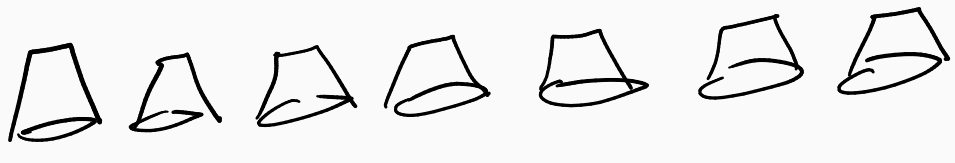
Analysing Kayles

N/P
labels

Grundy
values

Analysing Kyles

$$K_7 = \text{mex}(K_6, K_5, K_1 + K_5, K_1 + K_4, K_2 + K_4, K_2 + K_3, K_3 + K_3)$$

$K_7 =$ 

Analysing Kyles

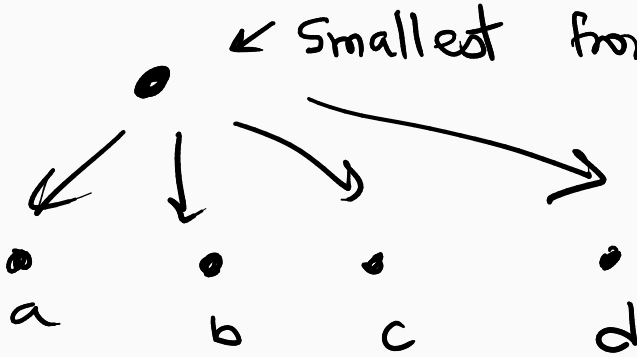
$K_7 = \text{mex}(K_6, K_5, K_1 + K_5, K_1 + K_4, K_2 + K_4, K_2 + K_3, K_3 + K_3)$

Better to just compute $K_1, K_2, K_3, \ldots$ in order.

Grundy labels:

$K_1, K_2, K_3, K_4, K_5, K_6$

(1)



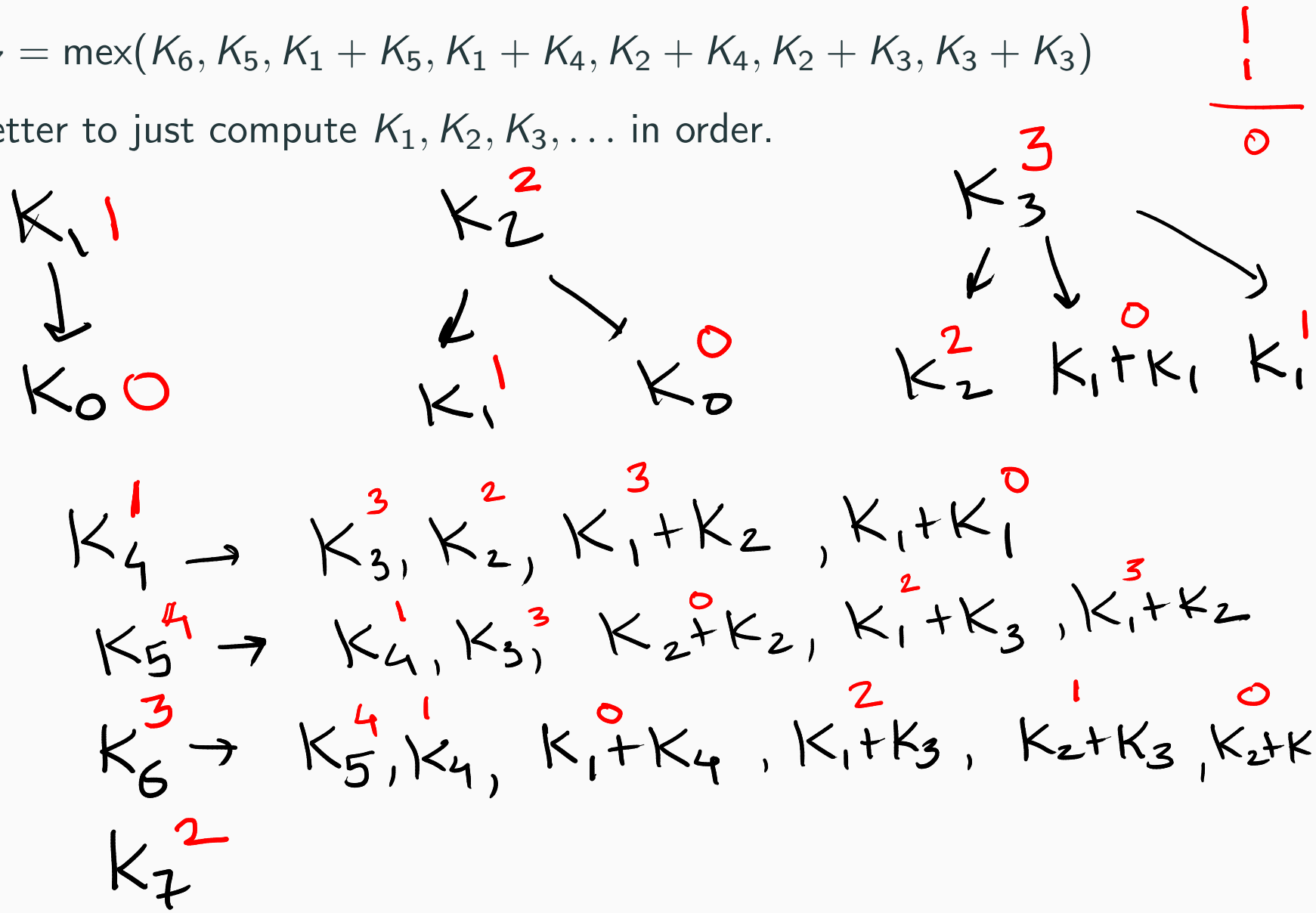
← smallest from 0,1,2, ... excluded from {a,b,c,d}

(2) $G + H \Rightarrow (G) \oplus (H)$

Analysing Kyles

$K_7 = \text{mex}(K_6, K_5, K_1 + K_5, K_1 + K_4, K_2 + K_4, K_2 + K_3, K_3 + K_3)$

Better to just compute $K_1, K_2, K_3, \ldots$ in order.



Now play and win!

What is known?

Kayles nim-values through K_{83}

K_n	0	1	2	3	4	5	6	7	8	9	10	11
0+	0	1	2	3	1	4	3	2	1	4	2	6
12+	4	1	2	7	1	4	3	2	1	4	6	7
24+	4	1	2	8	5	4	7	2	1	8	6	7
36+	4	1	2	3	1	4	7	2	1	8	2	7
48+	4	1	2	8	1	4	7	2	1	4	2	7
60+	4	1	2	8	1	4	7	2	1	8	6	7
72+	4	1	2	8	1	4	7	2	1	8	2	7