

# Games, graphs, and machines

## Partial orders 1

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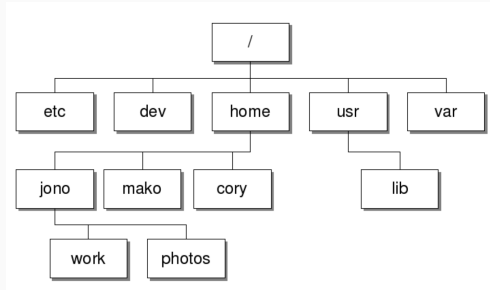
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# Partial orders

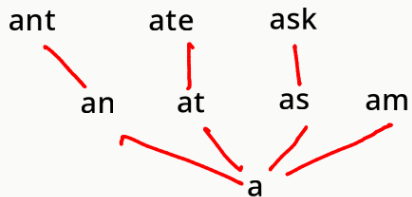
A *partial order* is a relation that is:

1. Reflexive
2. Anti-symmetric
3. Transitive

# Partial orders from real life



## Partial orders from real life



# Partial orders from real life



# Paper, Scissors, Rock: Partial order?

- Paper  $\preceq$  Scissors
- Scissors  $\preceq$  Rock
- Rock  $\preceq$  Paper

## Divisor poset

Let  $S$  be the set of divisors of 30; so  $S = \{1, 2, 3, 5, 6, 10, 15, 30\}$ .

Say  $a \preceq b$  if  $a$  divides  $b$ .

What is the Hasse diagram?

## Subset poset

Let  $S = \text{Pow}(\{1, 2, 3\})$ .

Say  $A \preceq B$  if  $A \subset B$ .

What is the Hasse diagram?

## String poset

Let  $S$  be the set of binary (0 and 1) strings of length three.

Say  $a_1a_2a_3 \preceq b_1b_2b_3$  if  $a_1 \leq b_1$  and  $a_2 \leq b_2$  and  $a_3 \leq b_3$ .

What is the Hasse diagram?